Path Tracing in Production

- part 1: modern path tracing
- part 2: making movies

Introduction (Johannes Hanika)
Motivation

continued effort to:

- document and discuss the switch of the movie industry to path tracing
- educate researchers, artists, programmers, .. about the particular requirements of making movies
- this year: fundamentals/basics/advanced techniques
  - glimpse at theoretical background
  - entry points for research
  - state of the art
  - toolset for improving rendering systems
- bring together industry experts
Speakers (part 1)

Johannes Hanika  Weta Digital, me
Luca Fascione    Weta Digital, Head of Technology and Research
Marc Droske      Weta Digital, Head of Rendering Research
Jorge Schwarzhaupt Weta Digital, Researcher + Manuka wizard
Christopher Kulla SPI, principal software engineer + Arnold veteran
Daniel Heckenberg Animal Logic, R&D Supervisor, ASWF chair
Course notes

What do you need to know to write better path tracers?
please see our course notes for a lot more detail!

Path tracing in Production

- part 1 and part 2
- part 2 follows in the afternoon
  - here, 403AB, 2pm-5:15pm
  - more about material acquisition, modelling, no Lions, and GPUs
Part 1: Modern path tracing

how to increase visual quality? (in order of direct relevance for movies)

- Physics! (follows now)
- Mathematics! (Luca)
- Designing an architecture for Monte Carlo! (Marc)
- Special sauce sampling! (Jorge)
- Smoke and explosions! (Chris)
- Mangling crazy complex input! (Daniel)
Physics!


Photons

- particle/wave dualism
  - we'll go with particles (for the most part), with a position and direction
- a **photon** corresponds to an atomic portion of energy $E$ (measured in Joule $[J]$)

$$E = \frac{h \cdot c_m}{\lambda} [J]$$

- where $h \approx 6.62607004 \times 10^{-34} [m^2 kg/s = Js]$ is Planck's constant,
- $c_m = c/\eta_m$ is the speed of light in the material with index of refraction $\eta_m$,
  and $c = 299,792,458 [m/s]$ is the speed of light in vacuum,
- and $\lambda [m]$ is the wavelength (often given in $[nm]$ instead to distinguish from world space lengths).

- we'll need a few ways to measure the energy of light
Phase space

- a photon lives in 5D **phase space**: 3D position $\mathbf{x}$ and 2D direction $\omega$

- positions are in **meters** [m]
- directions in **steradian** [sr]
  - dimensionless, derived SI unit of solid angle
Solid angle

- A direction is defined on the unit sphere, areas on this sphere are called solid angle.

\[ \Omega_A = \frac{A}{r^2} \text{ [sr]} \]

- As differential: \( d\omega \) or \( \sin \theta \, d\theta \, d\phi \) in polar form, since \( \int_\Omega d\omega = \int_0^{2\pi} \int_0^\pi \sin \theta \, d\theta \, d\phi = 4\pi \)

- We will refer to directions as \( \omega \) and \( \Omega = 4\pi \)
Radiance

- goal: want to measure light along a ray suitable for ray tracing
  - because we can only really evaluate visibility on a straight line between two points
  - spoiler:

\[
L(x, \omega) \left[ \frac{W}{m^2 \text{sr}} \right]
\]

- but first: let's count some photons!
Radiant energy

Count number $\#P$ of photons inside a certain volume, multiply by their energy $E$

$$Q = \#P \cdot \frac{h \cdot c_m}{\lambda} \left[ J = \frac{kg \cdot m^2}{s^2} \right]$$
Radiant power or flux

count photons inside certain volume per time (measured in watts):

\[ \Phi = \frac{dQ}{dt} \left[ \frac{J}{s} = W \right] \]
Ideally: measure light

- count photons per time in a volume $V$ in the 5D phase space over 3D positions and 2D directions $(x, \omega)$

$$\Phi = \int_{\Omega} \int_{V} P(x, \omega) \, dx \, d\omega \ [W]$$

- $P(x, \omega)$ is a symbol "counting" photons per time going through a certain point and direction in phase space

- note that $x$ is a 3D volume point here measured in cubic meters
Measure radiance

- power per area per solid angle (measured in watts per square meter per steradian)

\[ L(x, \omega) = \frac{d\Phi}{dxd\omega} \left[ \frac{W}{m^2 \text{sr}} \right] \]

- the central unit for us in rendering
- this is what a ray of light transports
- differentially small measurement apparatus: area with funnel
Measure radiance?

- dynamic system, light is in flow
- describe changes of radiance in phase space and solve for steady-state equilibrium!
Losses and gains

- three effects change the observed amount of light:
  - **collision**: light interacts with matter
  - **emission**: light is emitted from matter
  - **streaming**: light enters or leaves a volume

- collision may incur **scattering**: change of direction
The radiative transfer equation (RTE)

summing all terms

- all terms need to sum to zero (energy conservation)

\[
0 = \int_{\Omega} \int_{V} \left( -\frac{\partial}{\partial \omega} L(x, \omega) + \mu_e(x) L_e(x, \omega) - \mu_t(x) L(x, \omega) + \mu_s(x) \int_{\Omega} f_s(\omega_i \cdot \omega) L(x, \omega_i) d\omega_i \, dx \, d\omega \right)
\]

- if the equations hold for integration over any part of phase space, they have to hold for every individual point, too

- leave away integration over phase space \( \Omega \times V \)
The radiative transfer equation (RTE) 

All terms need to sum to zero (energy conservation), lose phase space integral

\[
\frac{\partial}{\partial \omega} L(x, \omega) = \mu_e(x)L_e(x, \omega) - \mu_t(x)L(x, \omega) + \mu_s(x) \int_\Omega f_s(\omega_i \cdot \omega)L(x, \omega_i)d\omega_i
\]

we are interested in changes in radiance \( L(x, \omega) \)
the directional derivative \( \frac{\partial}{\partial \omega} L(x, \omega) \) (LHS) is change of radiance in direction \( \omega \)
The radiative transfer equation (RTE)

summing all terms

\[ \frac{\partial}{\partial \omega} L(x, \omega) = \mu_e(x) L_e(x, \omega) - \mu_t(x) L(x, \omega) \]

\[ + \mu_s(x) \int_{\Omega} f_s(\omega_i \cdot \omega) L(x, \omega_i) d\omega_i \]

- integro differential equation along one ray \((x, \omega)\)
- governs scattering in participating media (volumes)
- we need to bring it to pure integral form for graphics applications!
Integrating the RTE

- example: solution of simple one-dimensional differential equation

\[
\frac{d}{dx} L(x) = -\mu_a L(x) \text{ with } x > 0, \; L(0) = 1
\]

- closed-form solution using the exponential function:

\[
L(x) = \exp(-\mu_a \cdot x) \cdot L(0)
\]

- gives rise to the transmittance term \(T(x, y)\)
The RTE in integral form

Integrating the RTE: intuition

- contribution from surface emission (exactly one point \( y \))

\[
L(x, \omega) \; {\pm} \; T(x, y) \cdot L_e(y, \omega)
\]
The RTE in integral form

Integrating the RTE: intuition

contributions from surface scattering (exactly one point \( y \))

\[
L(x, \omega) = T(x, y) \cdot \int_{\Omega} f_r(\omega_i, y, \omega) L(y, \omega_i) d\omega_i^\perp
\]
The RTE in integral form

Integrating the RTE: intuition

- contribution from volume emission (need to collect all \( z \))

\[
L(x, \omega) \quad + \quad T(x, z) \cdot \mu_e(z)L_e(z, \omega)
\]
The RTE in integral form

Integrating the RTE: intuition

Contribution from volume scattering (need to collect all $z$)

\[
L(x, \omega) \equiv T(x, z) \cdot \mu_s(z) \int \Omega f_s(\omega \cdot \omega_i) L(z, \omega_i) d\omega_i
\]
The RTE in integral form

\[ L(x, \omega) = T(x, y) \left( L_e(y, \omega) + \int_{\Omega} f_r(\omega_i, y, \omega) L(y, \omega_i) d\omega_i \right) \]

\[ + \int_0^d T(x, z) \left( \mu_e(z) L_e(z, \omega) + \mu_s(z) \int_{\Omega} f_s(\omega \cdot \omega_i) L(z, \omega_i) d\omega_i \right) dt \]

contribution from the point \( y \) on surface

contribution from any point \( z \) at distance \( t \) in volume
Recursive rendering equation

- light is emission + transported light (either from surface or volume)
  \[ L = L_e + TL \]

- Neumann series (\(T\) is a linear operator):
  \[ L = (1 - T)^{-1}L_e = \sum_{i=0}^{\infty} T^i L_e \]

- turns recursion into sum over all path lengths!
Tracing paths the recursive way

means tracing rays, usually through every pixel
Tracing paths the recursive way

- means tracing rays, usually through every pixel

- recursive rendering equation: light is emission + transported light

\[ L = L_e + TL \]
The rendering equation in path integral form

expand using the Neumann series to arrive at path space

\[ I_p = \int_{\mathcal{P}} h_p(\mathbf{X}) \cdot f(\mathbf{X}) d\mathbf{X} \]

\( h_p(\mathbf{X}) \) selects paths per pixel \( p \) via pixel filter support

\( \mathbf{X} = (x_1, x_2, \ldots x_k) \in \mathcal{P} \) is list of vertices \( x \)

\( f(\mathbf{X}) \) is the **measurement contribution function** in product vertex area measure \( d\mathbf{X} \)
The measurement contribution function

- measure differential power of path $\mathbf{X}$ with length $k$ (\(= 5\) vertices here)

$$f(\mathbf{X}) = L_e G_{k-1} \left( \prod_{i=1}^{k-2} f_{r,i} G_i \right) W$$
The measurement contribution function

measure differential power of path $\mathbf{X}$ with length $k$ (≈ 5 vertices here)

$$f(\mathbf{X}) = \frac{d^k \Phi}{d^k \mathbf{x}} \left[ \frac{W}{m^{2 \cdot k}} \right]$$
Monte Carlo integration

- approximate the integral by a Monte Carlo estimator

\[ I_p \approx \frac{1}{N} \sum_{i=1}^{N} \frac{h_p(X_i) \cdot f(X_i)}{p(X_i)} \]

- the expected value of the estimator is precisely the integral (the estimator is unbiased)
- error manifests itself as noise (variance of the estimator)
- how much noise for which path construction strategy determined by their PDF \( p(X) \)
Path tracing

- start constructing a path at the sensor
- sample outgoing direction locally by Bsd
Path tracing

- start constructing a path at the sensor
- problem intersecting light source by chance
Path tracing/next event estimation

- start constructing a path at the sensor
- direct connection(s) to light source in area measure
Path tracing/next event estimation

- problem connecting glossy/specular materials
- Bsdf evaluates to zero (or close to for low roughness > 0)
Light tracing

- reverse the tracing direction, start at the light sources
- good for caustics
Light tracing

- but doesn't work for specular, either
- SDS doesn't work even for combination of all the above techniques (called BDPT)
Vertex connection and merging (VCM/UPS)

- uses photon maps to cover SDS paths
- expensive: big storage, long kd-tree build times, many combinations to evaluate
Metropolis light transport (MLT)

- mutates initial sample, uses current path as Markov chain state
- often leads to temporal inconsistency/blotches
Program part 1:

- 09:00 — Opening statement and introduction to path tracing (almost over now!)  
  (30 min, Johannes Hanika)
- 09:30 — A short History of Monte Carlo  
  (30 min, Luca Fascione)
- 10:00 — Implementing path sampling techniques  
  (30 min, Marc Droske)
- 10:30 — Break (15 min)
Program part 1, cntd:

- 10:45 — Finding good paths
  (30 min, Jorge Schwarzhaupt)
- 11:15 — Volumes
  (30 min, Christopher Kulla)
- 11:45 — The Ins of Production Rendering at Animal Logic
  (30 min, Daniel Heckenberg)
Path tracing in production, part 2
will continue in the afternoon session

here, 403AB, 2pm-5:15pm
Let's get started!
Thank you for listening!

questions?

please find our course notes here: