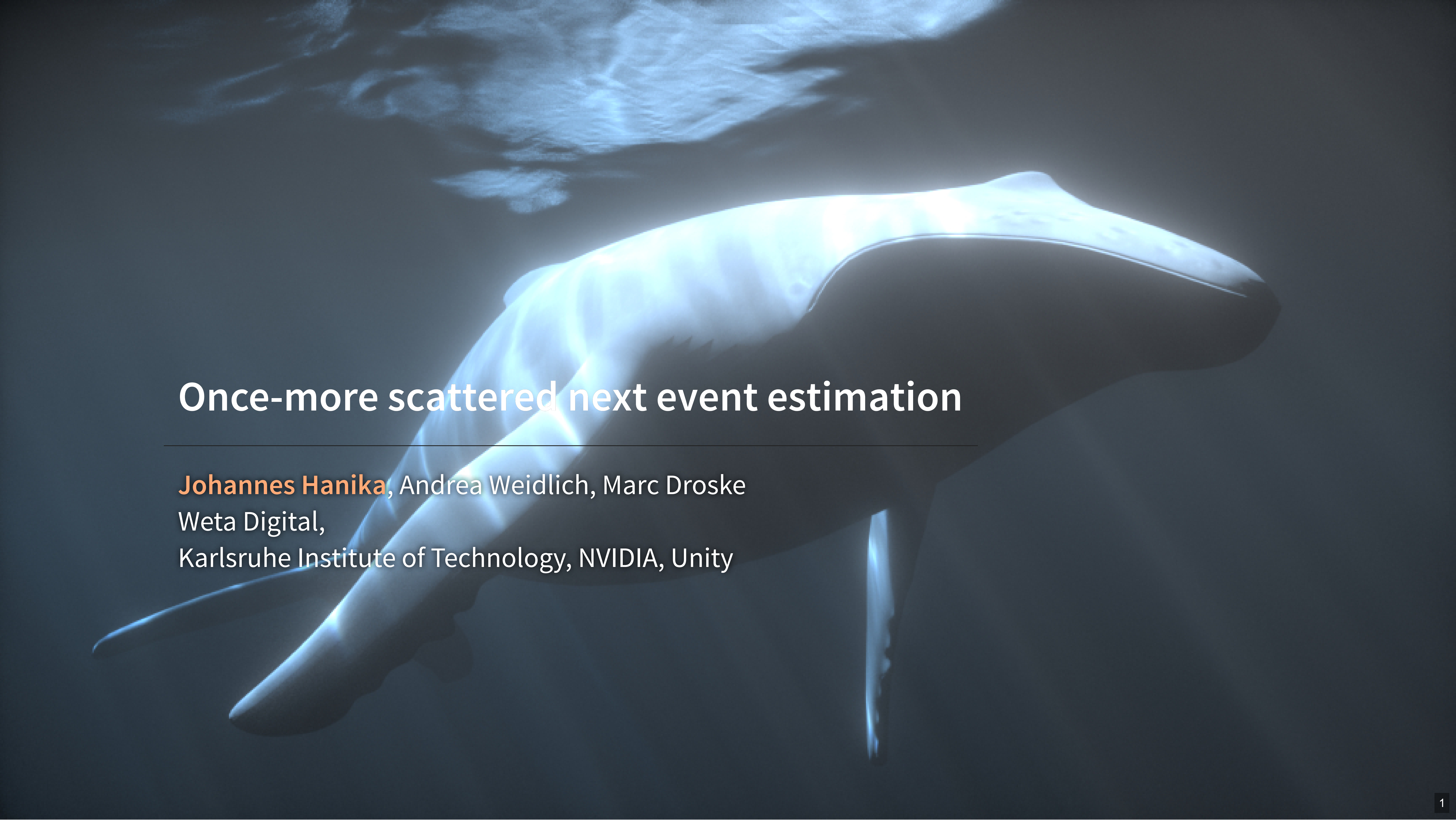

Once-more scattered next event estimation

Johannes Hanika, Andrea Weidlich, Marc Droske
Weta Digital,
Karlsruhe Institute of Technology, NVIDIA, Unity


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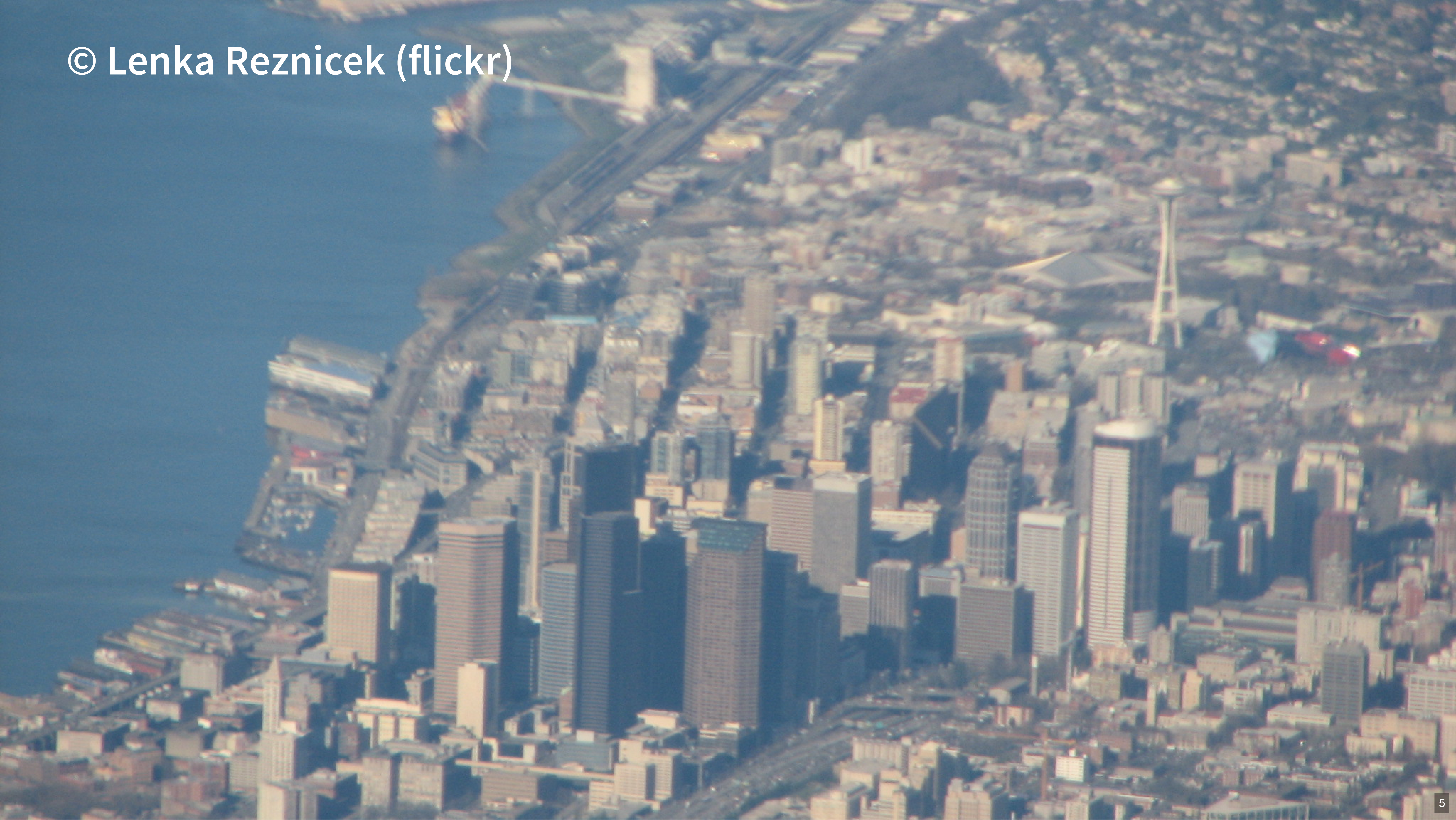
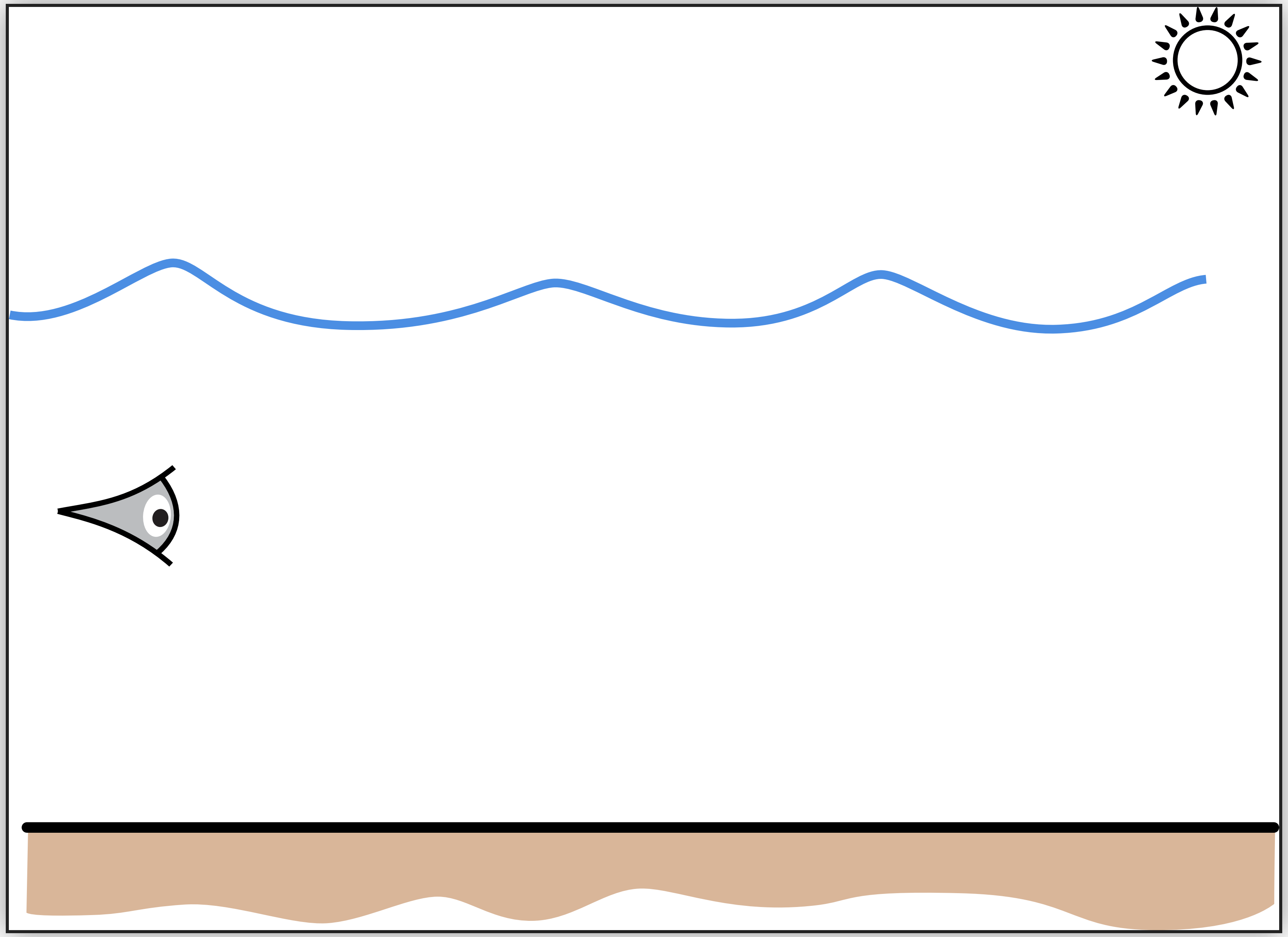


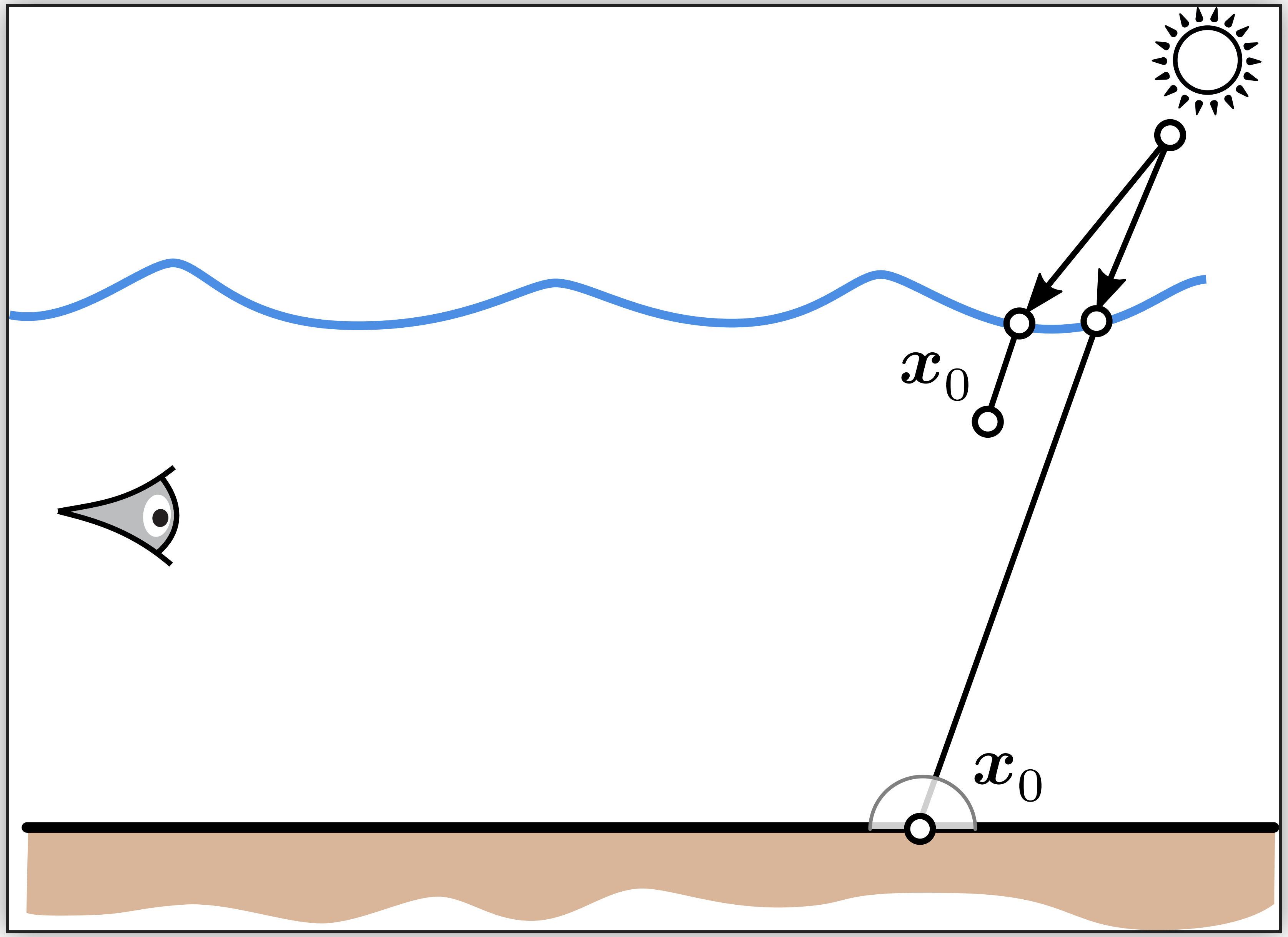
Which transport paths create this effect?

- Subtle blurring with depth
- Let's take the underwater example:



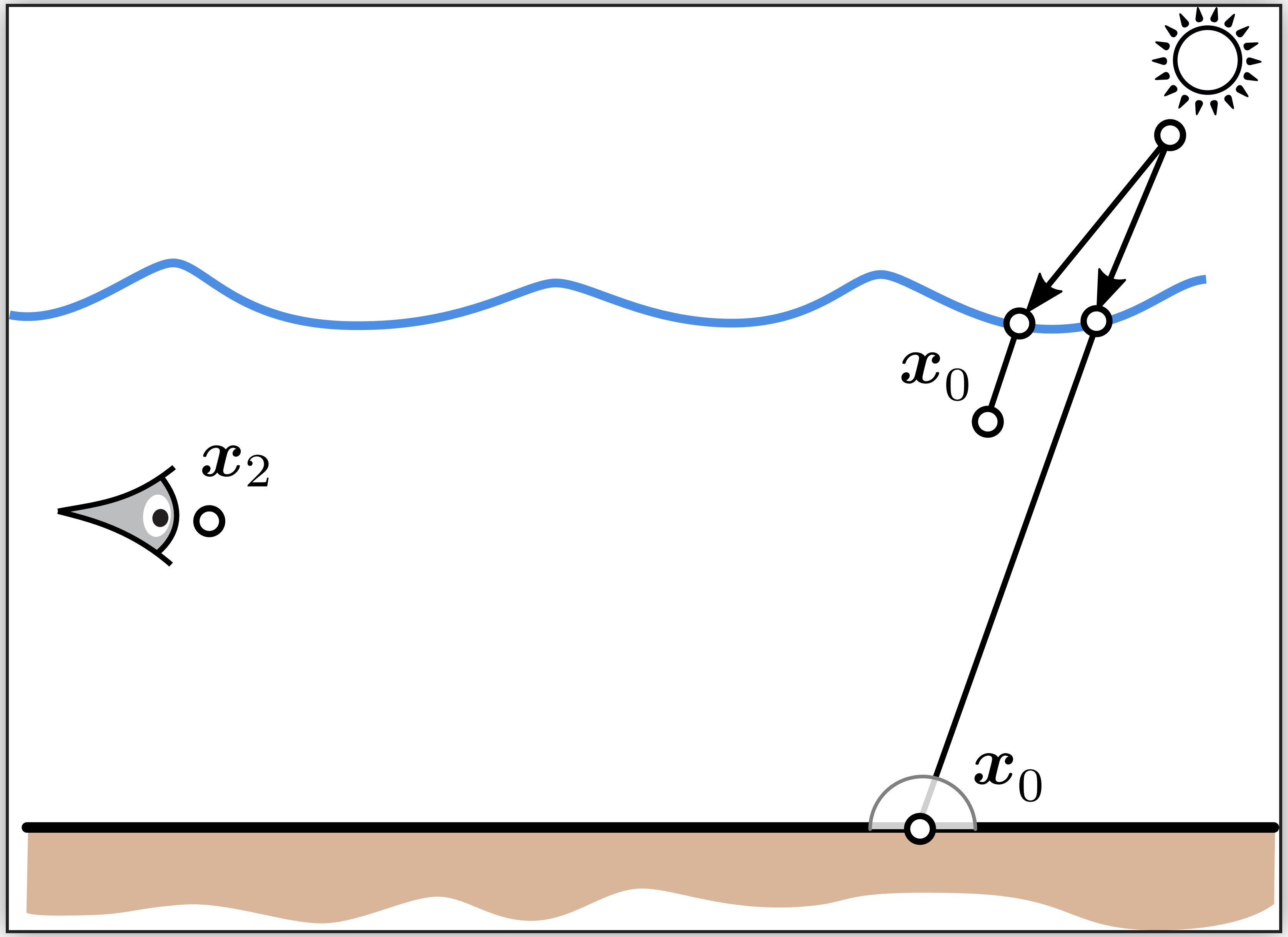
Which transport paths create this effect?

- Let's take the underwater example:
- Light travels from the sun to under water



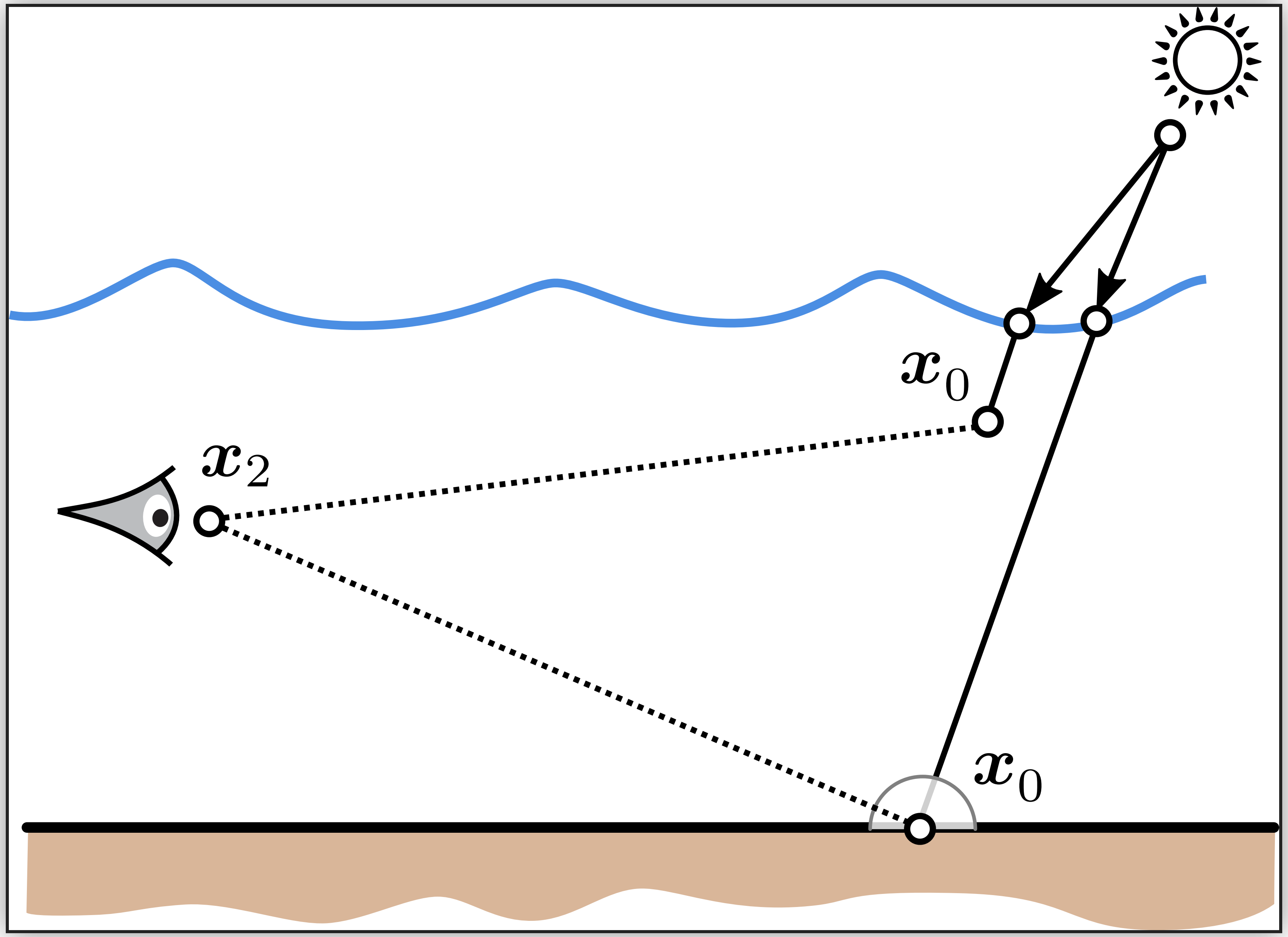
Which transport paths create this effect?

- At each endpoint $\boldsymbol{x}_0$:
- How do we connect endpoints to $\boldsymbol{x}_2$ on the eye?



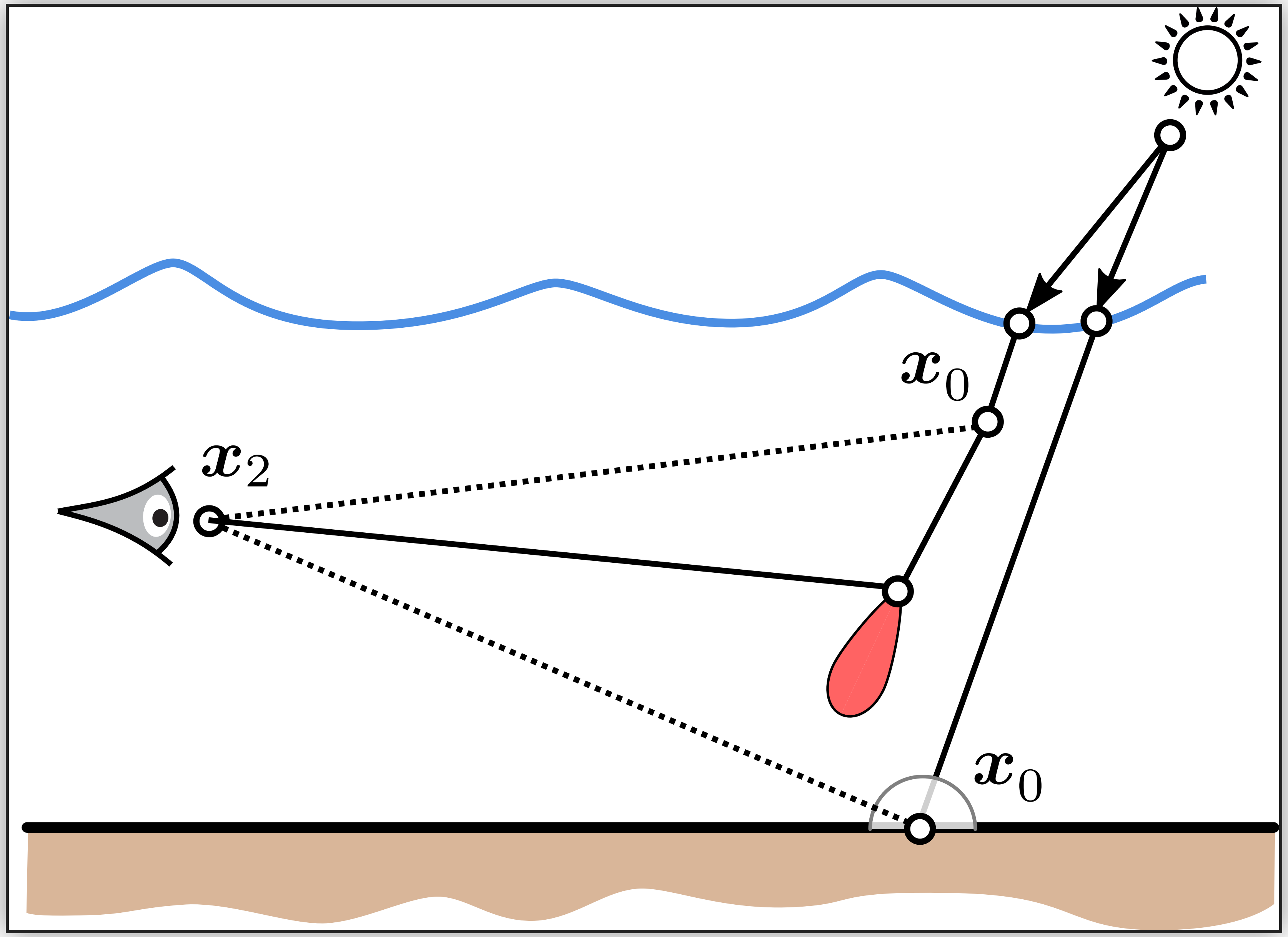
Which transport paths create this effect?

- Classic next event estimation (NEE)?
- Results in sharp images, no blur!



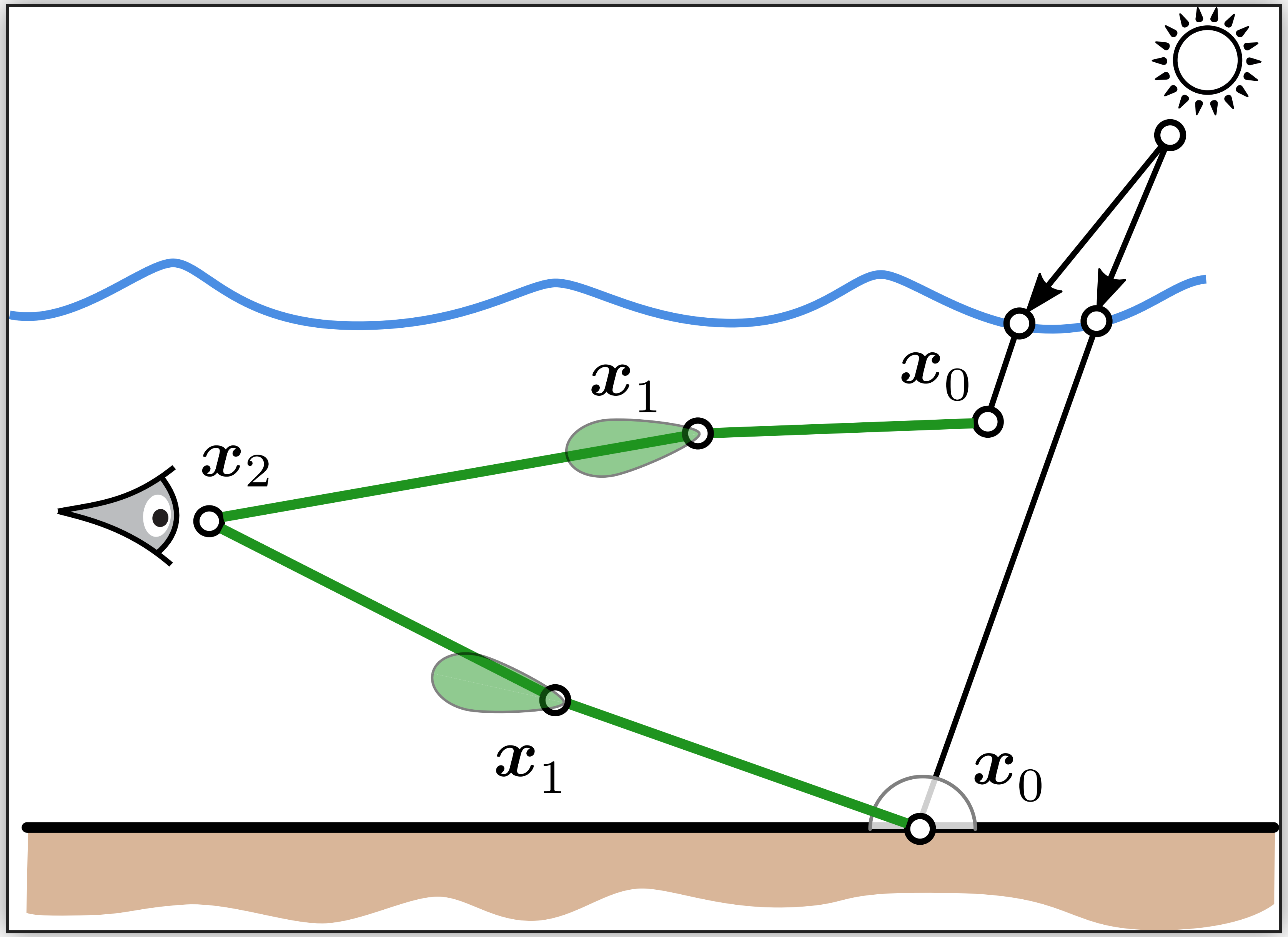
Which transport paths create this effect?

- Extend the path once more before NEE?
- Also important, but visually different feature



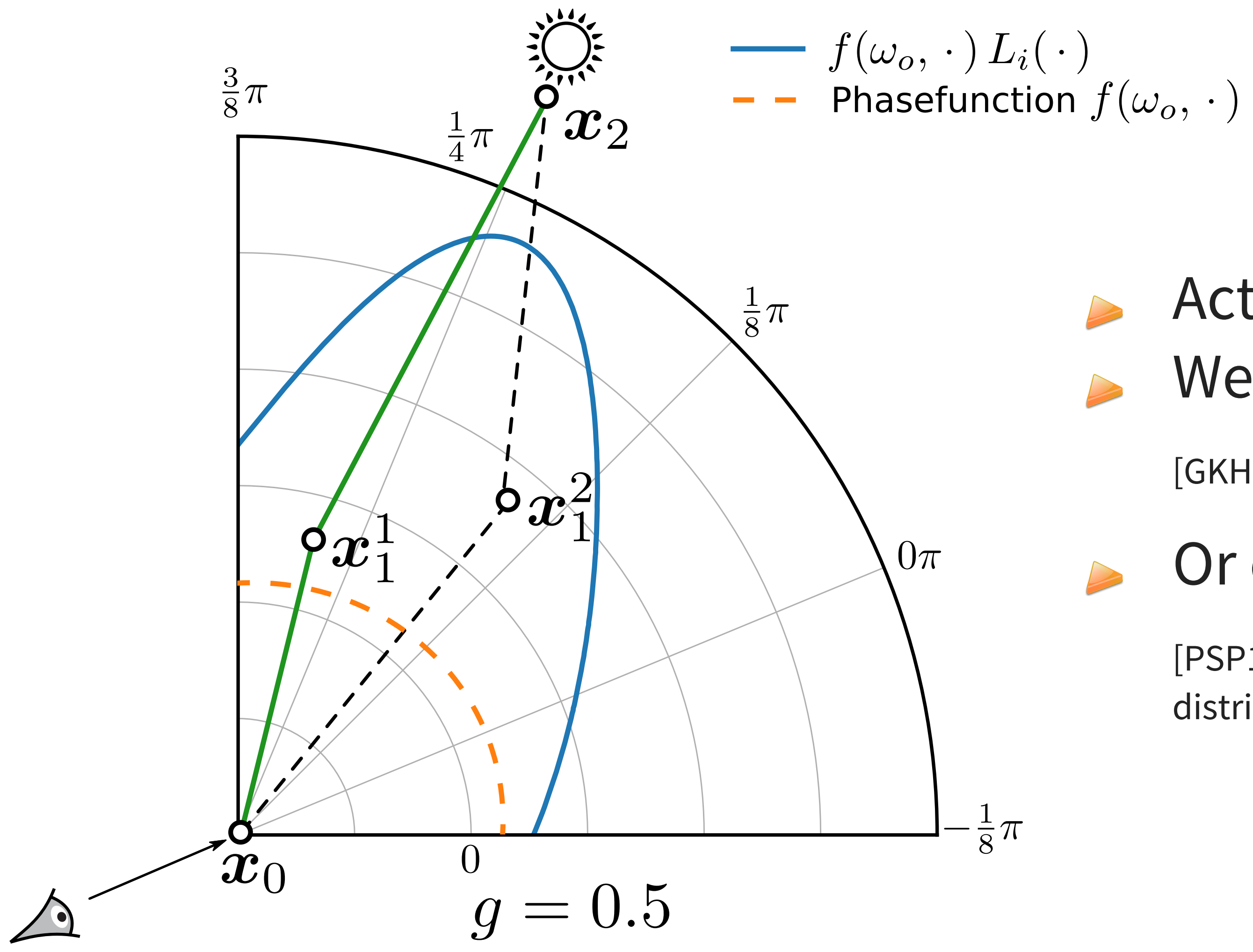
Which transport paths create this effect?

- Achieve characteristic volumetric blur:
- Need to sample phase function at $\boldsymbol{x}_1$!



How different are the two effects?

for moderately forward scattering phase function



- Actually the same effect (product of light and phase function)
- We know how to sample it: via joint importance sampling/tabulation

  [GKH*13] Georgiev et al., “Joint importance sampling of low-order volumetric scattering”, SIGGRAPH Asia 2013.

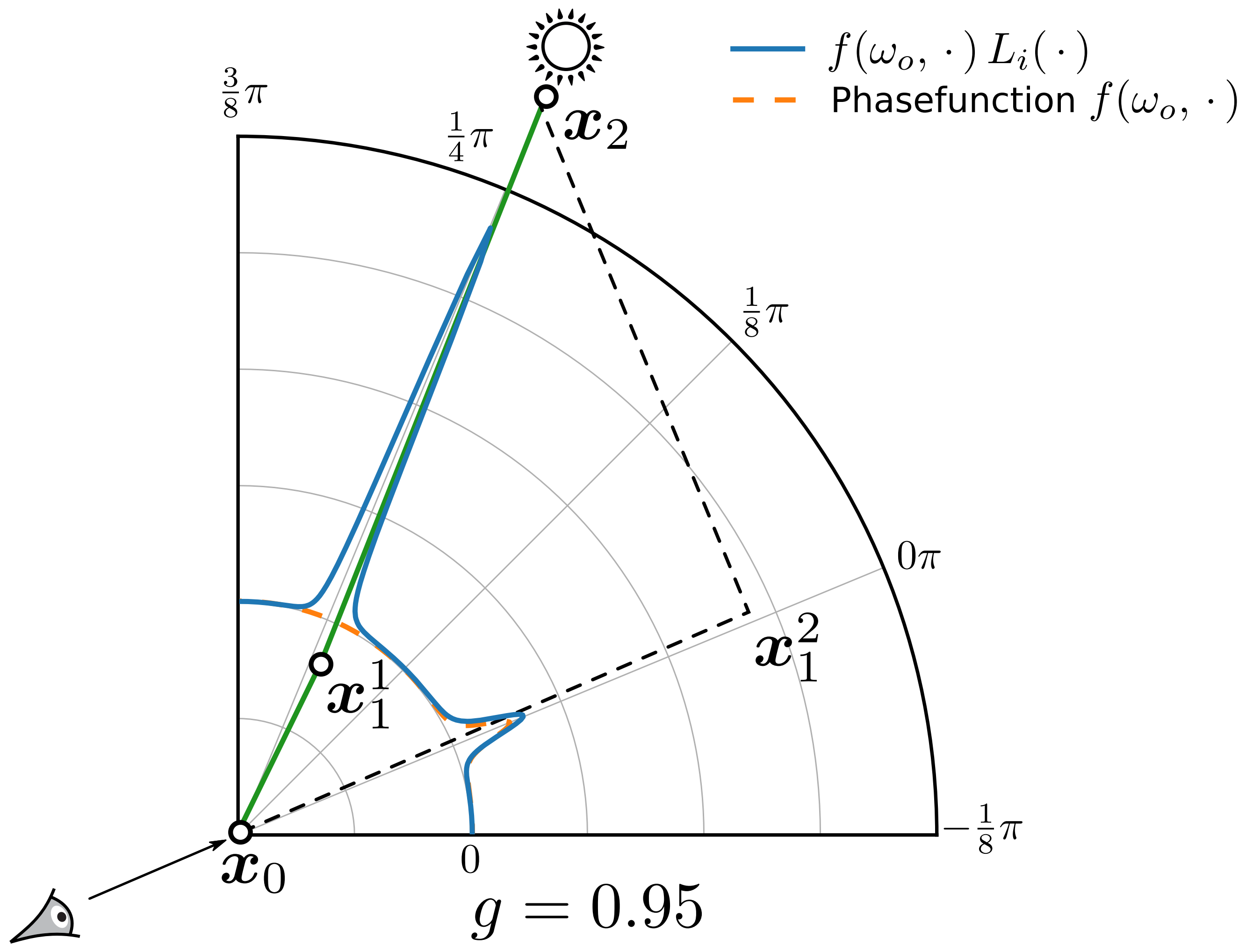
- Or evaluate analytically via series expansion of the phase function

  [PSP10] Pegoraro et al., “A closed-form solution to single scattering for general phase functions and light distributions”, EGSR 2010.

- Blue line: unit test collected histogram of contributions over all $\boldsymbol{x}_1$, plotted over outgoing angle, contains $L_i \cdot f_s$

How different are the two effects?

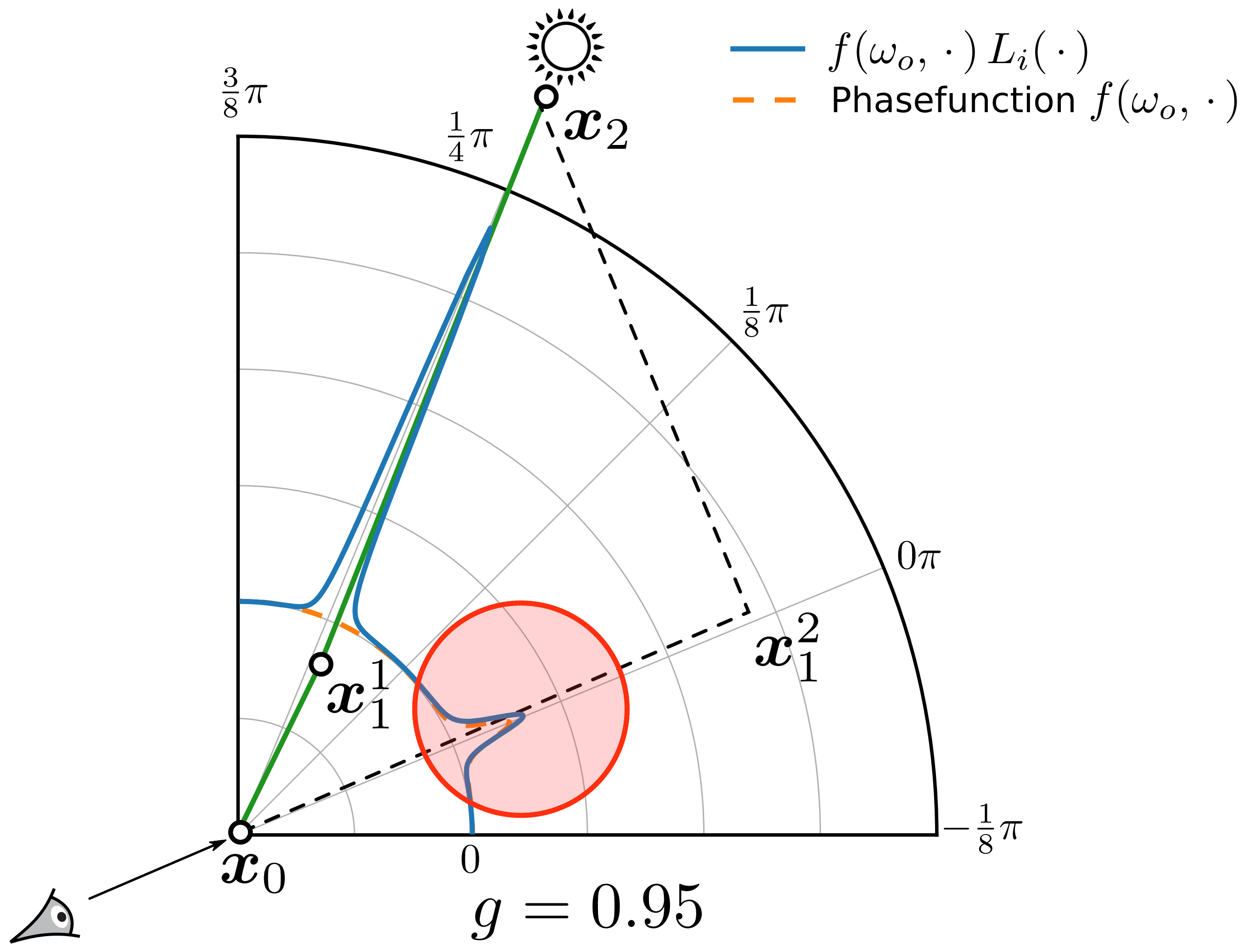
for highly forward scattering phase function



very!

How different are the two effects?

for highly forward scattering phase function

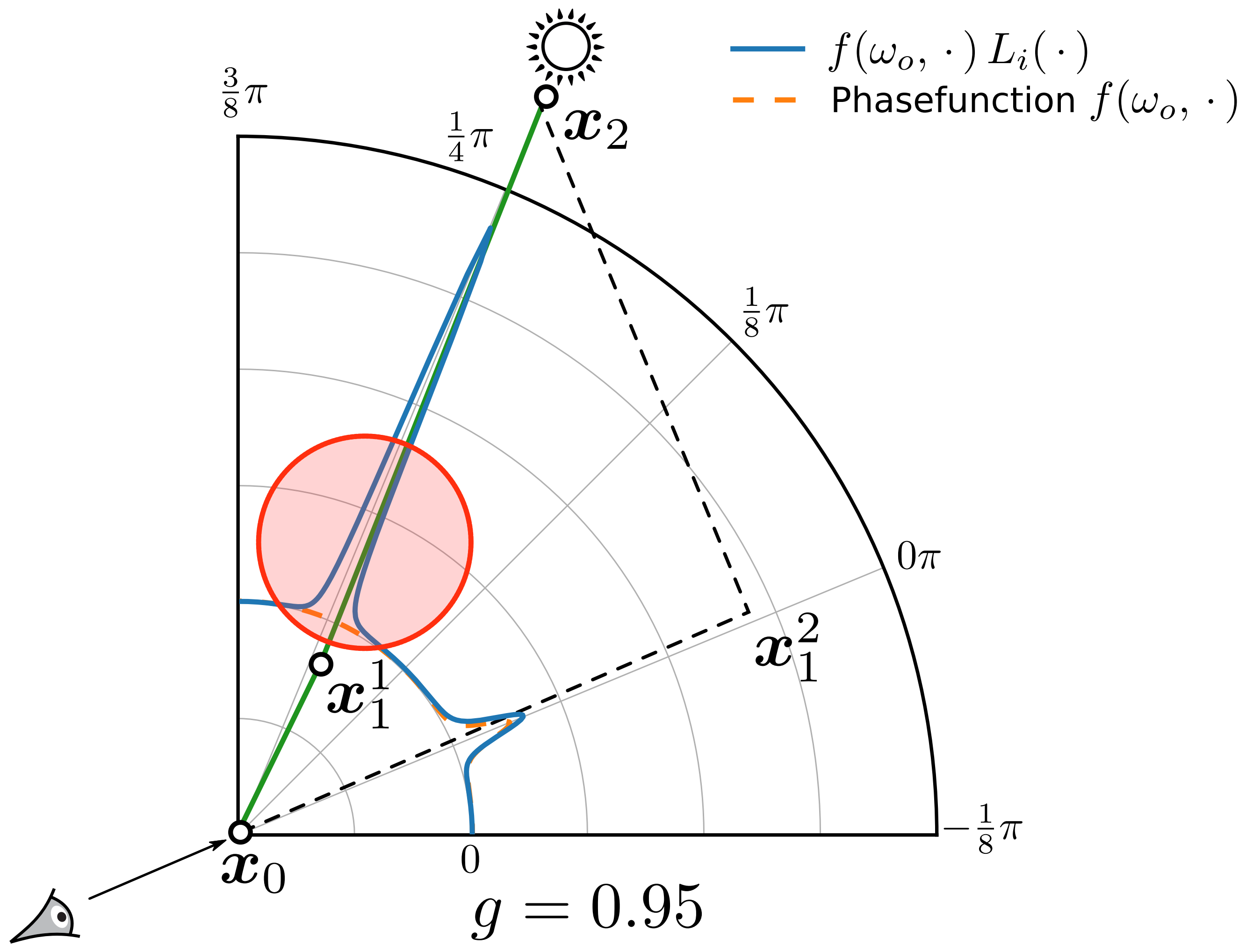


- Can sample phase function (marked peak), together with equiangular for best connection

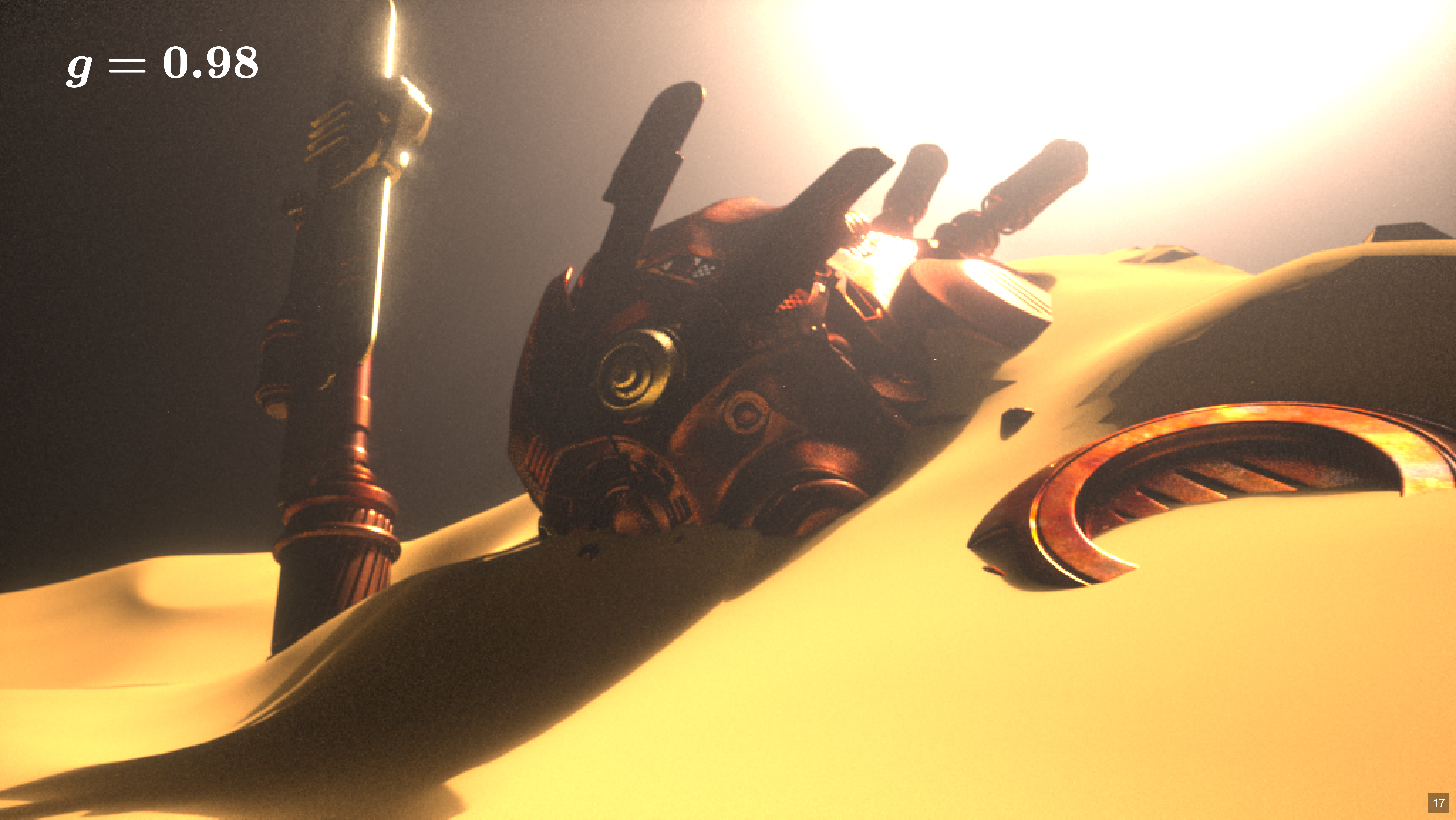
[KF12] Kulla and Fajardo, “Importance sampling techniques for path tracing in participating media”, EGSR 2012.

How different are the two effects?

for highly forward scattering phase function



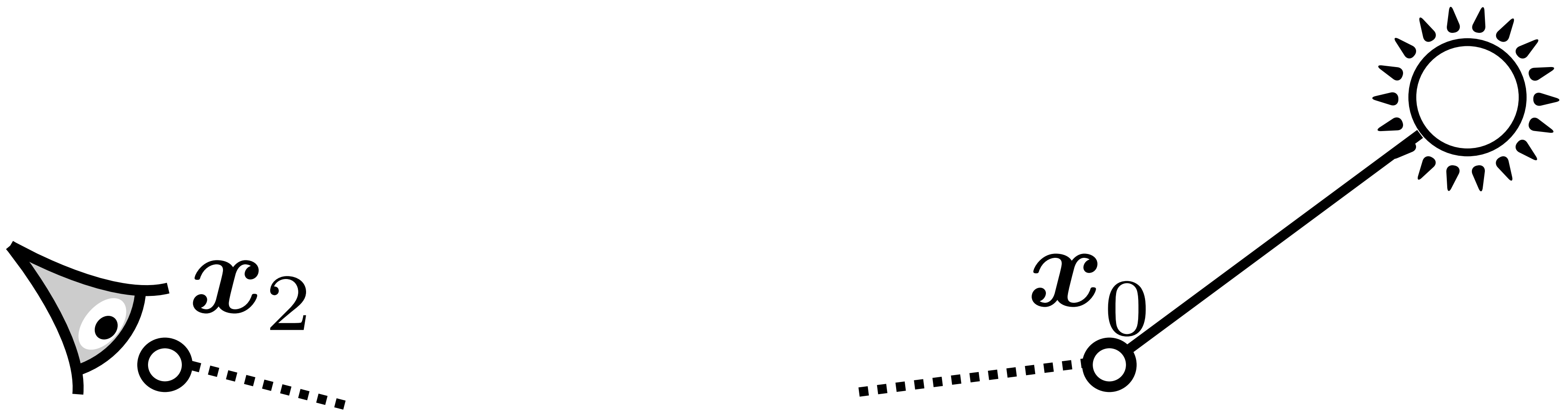
- Cannot sample the other lobe!
- This is what we'll do in the remainder of this talk!
- But is that visually important? What does not-so-peaky look like?


$g = 0.98$

$g = 0.5$

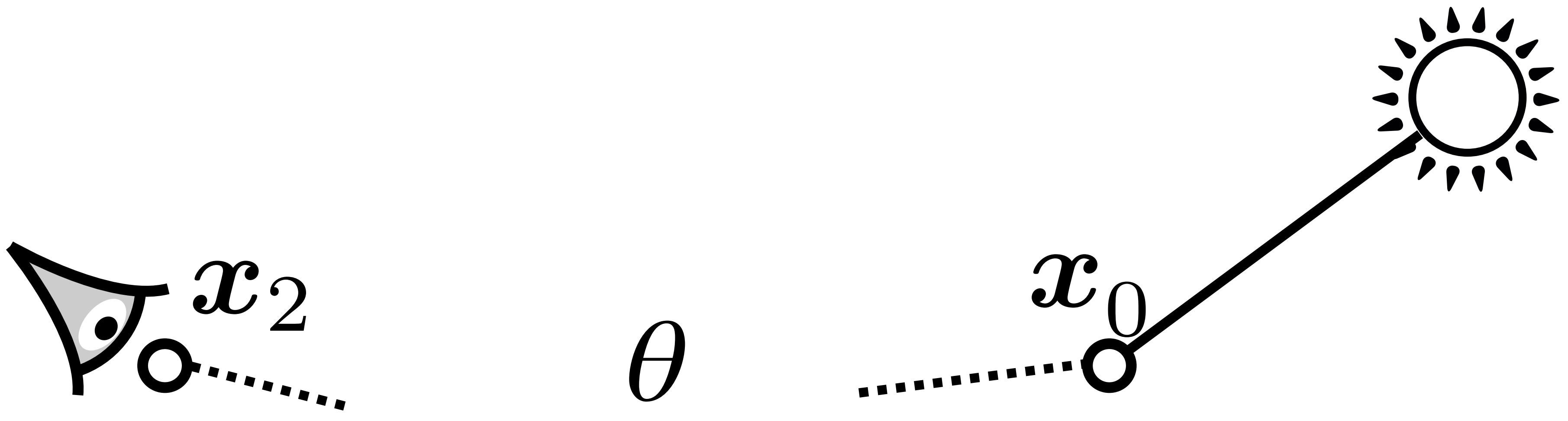
Phase function is most important here!

- Why don't we sample it first?



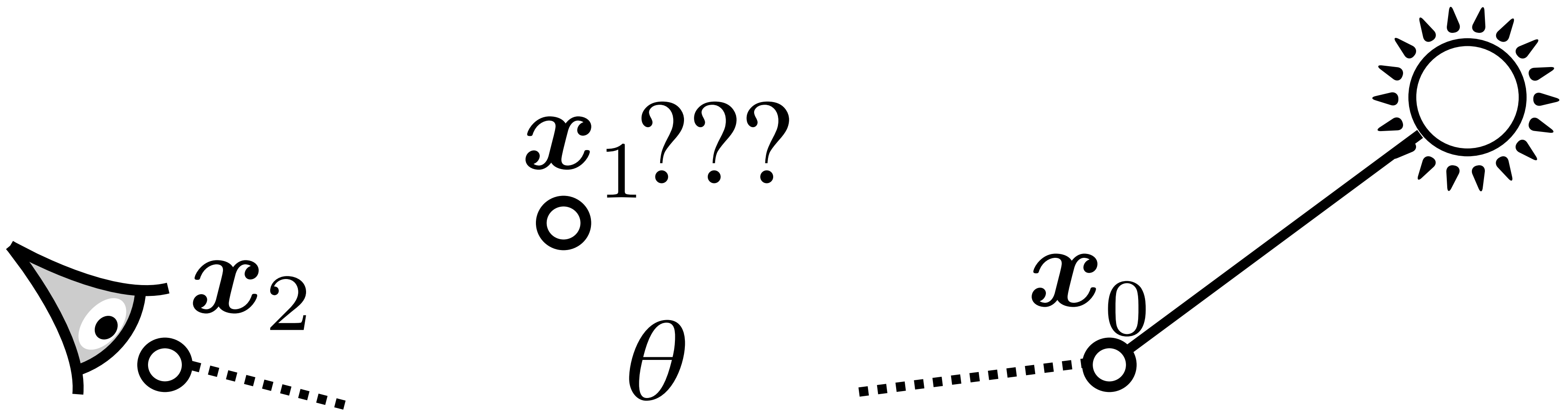
Phase function is most important here!

- Why don't we sample it first: fix θ

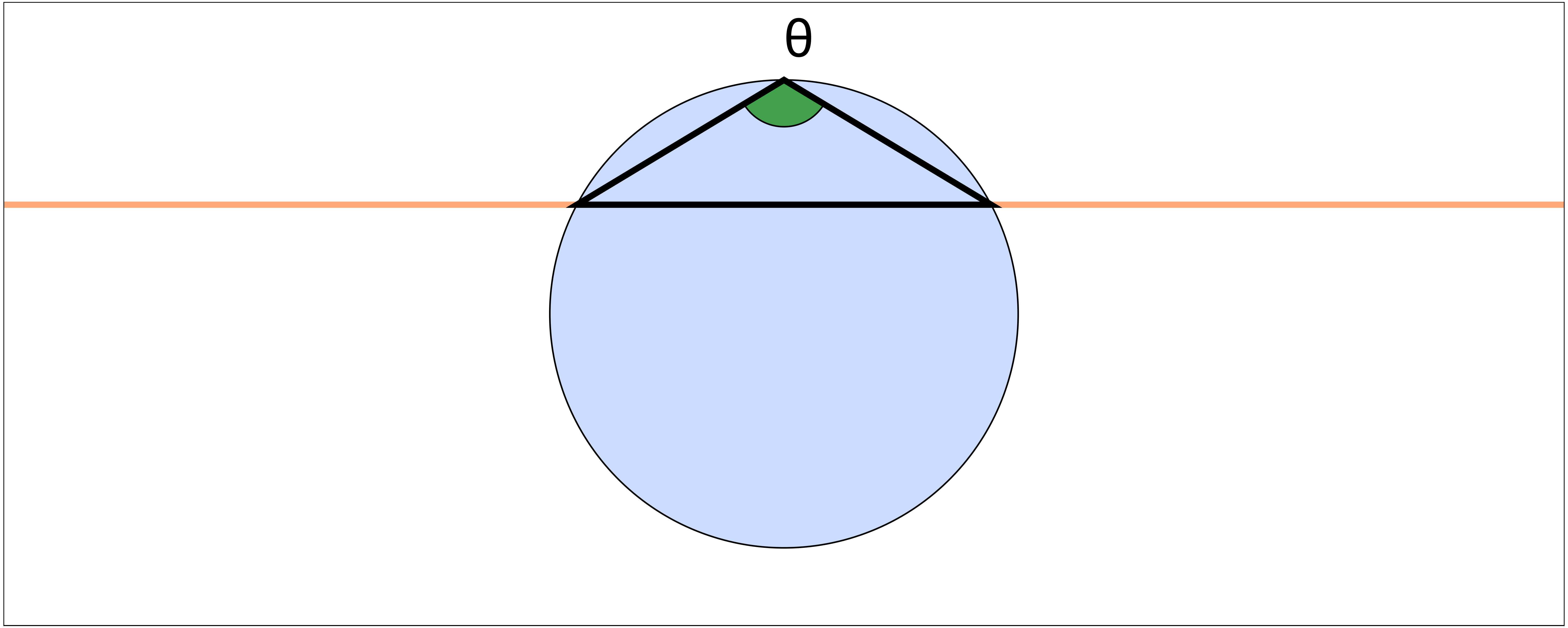


Phase function is most important here!

- Where to place $\boldsymbol{x}_1$ then?



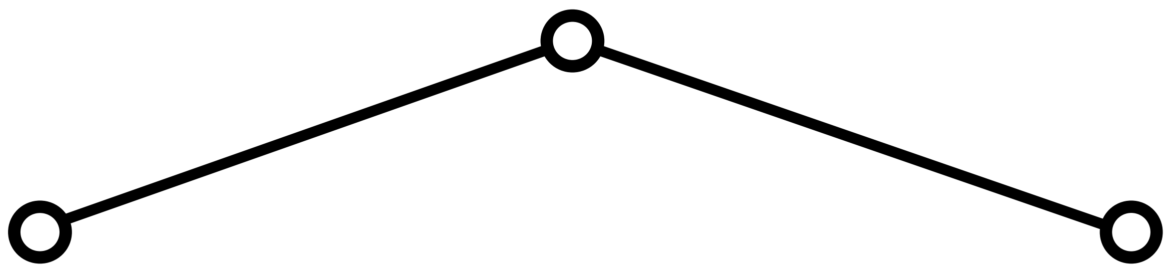
Circumcircle geometry



- "Waterlevel" in the demo determines θ at center vertex of triangle
- Picking any $\boldsymbol{x}_1$ on the arc (above waterlevel) has constant θ!

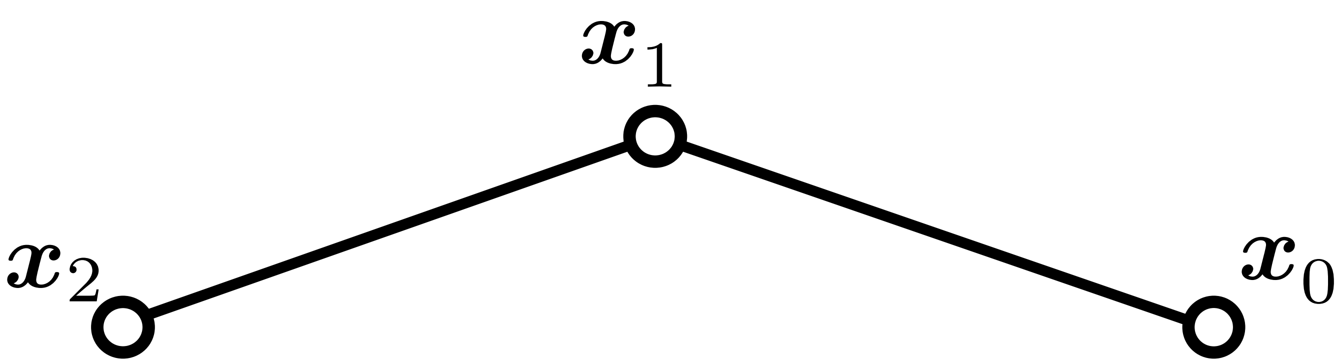
Circumcircle geometry

A few definitions



Circumcircle geometry

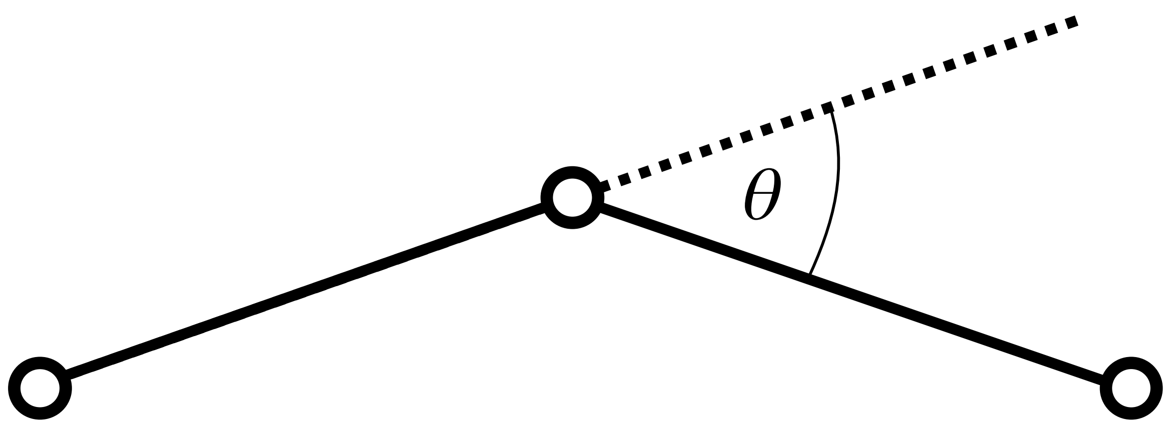
A few definitions



- Corner vertices $\boldsymbol{x}$.

Circumcircle geometry

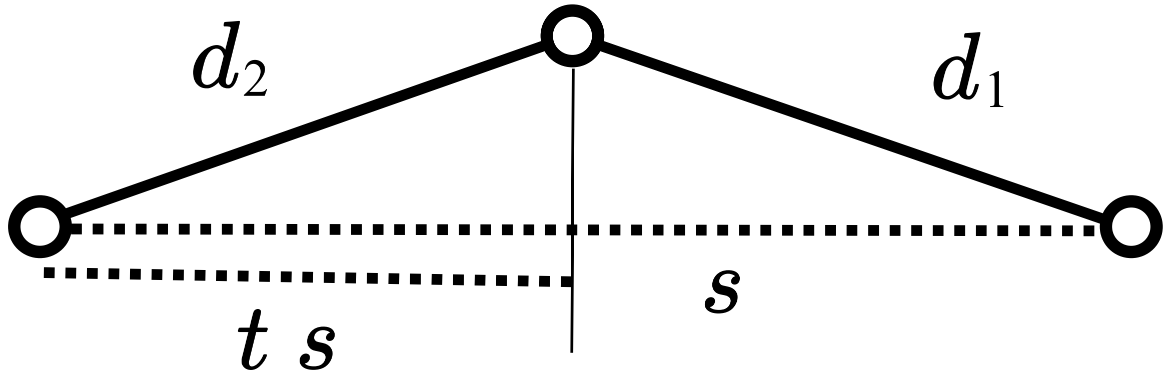
A few definitions



- The phase function angle θ

Circumcircle geometry

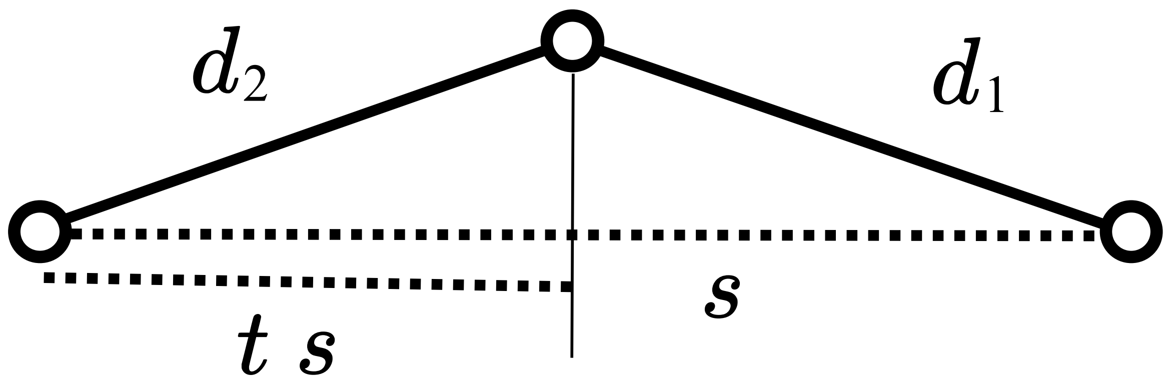
A few definitions



- Triangle edges: d_1, d_2, s
- t is the fractional distance between $\boldsymbol{x}_0$ and $\boldsymbol{x}_2$
- $t \in [0, 1]$ means $\theta < \pi/2$ means forward scattering only!

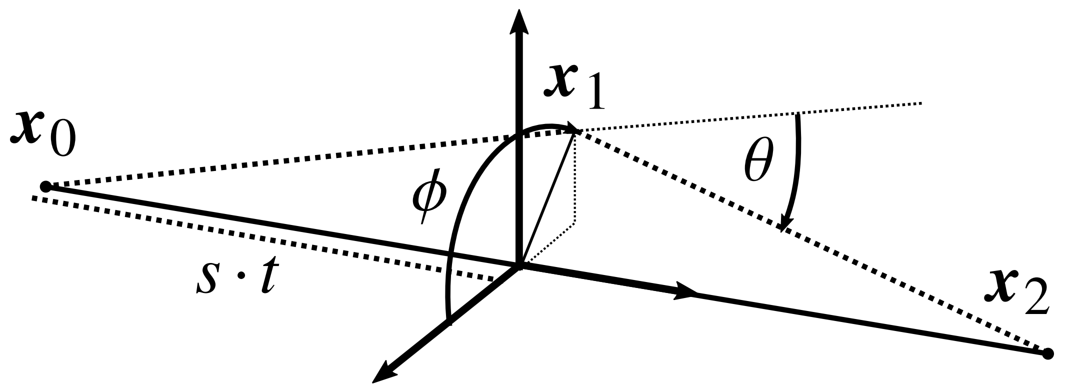
Circumcircle geometry

A few definitions



- Sample t, then find $\boldsymbol{x}_1$ on the arc!
- Pick t such that the geometry terms $\frac{1}{d_1^2 \cdot d_2^2}$ cancel

A change of variables



- Need to integrate all flux via any $\boldsymbol{x}_1$

$$I(\boldsymbol{x}_0 \leftrightarrow \boldsymbol{x}_2) = \int_{\boldsymbol{x}_1} f(\boldsymbol{x}_0 \leftrightarrow \boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2) \mathrm{d}\boldsymbol{x}_1$$

- Parameterise $\boldsymbol{x}_1$ in 3D via θ, ϕ, t
- Your everyday Monte Carlo change of variables/Jacobian computation, let's do it!

A few pages of maths later..

$$R(\theta) = \frac{\overline{\boldsymbol{x}_0\boldsymbol{x}_2}}{2s\sin(\pi-\theta)} = \frac{1}{2\sin(\theta)}. \quad (10)$$

From this we can compute the normalised polar radius r as

$$r(t,R) = \sqrt{R^2-(1/2-t)^2} - \sqrt{R^2-1/4}, \quad (11)$$

$$r(t,\theta) = \sqrt{\frac{1}{4\sin^2\theta}-(1/2-t)^2} - \sqrt{\frac{1}{4\sin^2\theta}-1/4}. \quad (12)$$

As another intermediate step, we will next go to regular spherical coordinates $(\hat{r},\hat{\theta},\hat{\phi}=\phi)$ (see Figure 7):

$$\int_S f\,\mathrm{d}\mathbf{x} = \int_{\hat{\phi}}\int_{\hat{\theta}}\int_{\hat{r}} f\cdot\left|\frac{\partial\mathbf{x}}{\partial(\hat{\theta},\hat{r},\hat{\phi})}\right|\mathrm{d}\hat{r}\,\mathrm{d}\hat{\theta}\,\mathrm{d}\hat{\phi} \quad (13)$$

$$= \int_{\hat{\phi}}\int_{\hat{\theta}}\int_{\hat{r}} f\cdot|\hat{r}^2\sin\hat{\theta}|\,\mathrm{d}\hat{r}\,\mathrm{d}\hat{\theta}\,\mathrm{d}\hat{\phi}. \quad (14)$$

$$|J| = \left|\frac{\partial(\theta,t)}{\partial(\hat{r},\hat{\theta})}\right| = \left|\frac{\partial\theta}{\partial\hat{r}}\cdot\frac{\partial t}{\partial\hat{\theta}} - \frac{\partial\theta}{\partial\hat{\theta}}\cdot\frac{\partial t}{\partial\hat{r}}\right|. \quad (19)$$

The fractional distance t and its derivatives are simple:

$$t(\hat{r},\hat{\theta}) = \frac{\hat{r}}{s}\cos\hat{\theta}+1/2 \quad (20)$$

$$\partial t/\partial\hat{\theta} = -\frac{\hat{r}}{s}\sin\hat{\theta} \quad (21)$$

$$\partial t/\partial\hat{r} = \frac{1}{s}\cos\hat{\theta}. \quad (22)$$

$$d_2^2 = \hat{r}^2+\frac{s^2}{4}-\hat{r}s\cos\hat{\theta} \quad (23)$$

$$d_1^2 = \hat{r}^2+\frac{s^2}{4}-\hat{r}s\cos(\pi-\hat{\theta}) = \hat{r}^2+\frac{s^2}{4}+\hat{r}s\cos\hat{\theta}, \quad (24)$$

nd we also simplify the expression for their product

$$d_1^2\cdot d_2^2 = -\frac{16\hat{r}^2s^2\cos^2\hat{\theta}-s^4-8\hat{r}^2s^2-16\hat{r}^4}{16} \quad (25)$$

$$= -\frac{1}{16}\left(8\hat{r}^2s^2(2\cos^2\hat{\theta}-1)-16\hat{r}^4-s^4\right) \quad (26)$$

$$= \frac{1}{16}\left(16\hat{r}^4+s^4-8\hat{r}^2s^2\cos 2\hat{\theta}\right). \quad (27)$$

Now we are ready to compute θ from $\hat{\theta}$ and $\hat{r}$ as

$$\theta(\hat{r},\hat{\theta}) = \pi - \arccos\left(\frac{d_1^2+d_2^2-s^2}{2d_1d_2}\right) \quad (28)$$

$$= \pi - \arccos\left(\frac{\hat{r}^2-s^2/4}{d_1d_2}\right) \quad (29)$$

$$= \pi - \arccos\left(\frac{4\hat{r}^2-s^2}{\sqrt{16\hat{r}^4+s^4-8\hat{r}^2s^2\cos 2\hat{\theta}}}\right), \quad (30)$$

as well as the partial derivatives required for the Jacobian in Equation (19)

$$\partial\theta/\partial\hat{r} = \frac{4s(4\hat{r}^2+s^2)\sin\hat{\theta}}{16\hat{r}^4+s^4-8\hat{r}^2s^2\cos 2\hat{\theta}}, \quad (31)$$

$$\partial\theta/\partial\hat{\theta} = \frac{4\hat{r}s(s^2-4\hat{r}^2)\cos\hat{\theta}}{16\hat{r}^4+s^4-8\hat{r}^2s^2\cos 2\hat{\theta}}. \quad (32)$$

```
FullSimplify[Abs[4 rr Sin[hh]/(4 rr^2 Cos[2*hh] - s^2)] / Sin[theta] //.
  {
   hh → ArcSin[s r / Sqrt[s^2 (t - 1/2)^2 + s^2 r^2]],
   rr → Sqrt[s^2 (t - 1/2)^2 + s^2 r^2],
   r → Sqrt[1/(4 Sin[theta]^2) - (1/2 - t)^2] - Sqrt[1/(4 Sin[theta]^2) - 1/4]
  }]
```

Out[ ]= $\frac{2\,\mathrm{Csc[theta]}\left(-\mathrm{Cot[theta]}+\sqrt{-1+4t-4t^2+\mathrm{Csc[theta]}^2}\right)}{s\,\mathrm{Abs}\left[8(-1+t)t+2\,\mathrm{Cot[theta]}\left(-\mathrm{Cot[theta]}+\sqrt{-1+4t-4t^2+\mathrm{Csc[theta]}^2}\right)\right]}$

In[ ]:= TrigReduce[$\frac{2\,\mathrm{Csc[theta]}\left(-\mathrm{Cot[theta]}+\sqrt{-1+4t-4t^2+\mathrm{Csc[theta]}^2}\right)}{s\,\mathrm{Abs}\left[8(-1+t)t+2\,\mathrm{Cot[theta]}\left(-\mathrm{Cot[theta]}+\sqrt{-1+4t-4t^2+\mathrm{Csc[theta]}^2}\right)\right]}$]

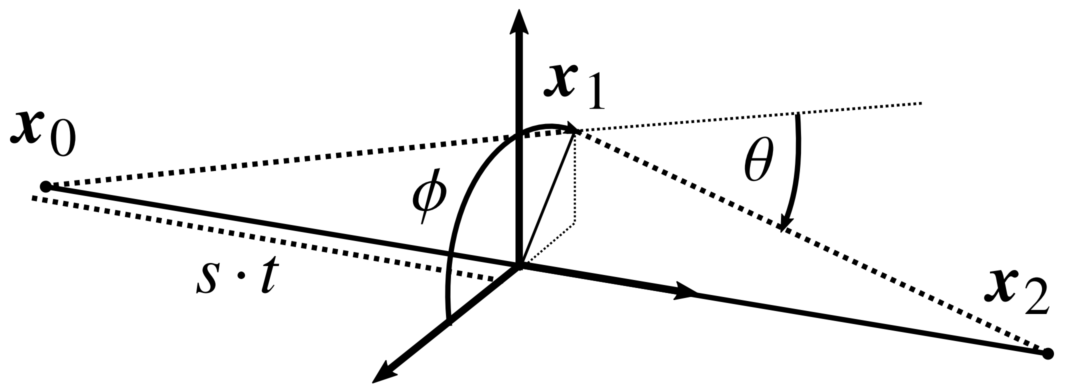
Out[ ]= $-\frac{2\,\mathrm{Csc[theta]}\left(\mathrm{Cot[theta]}-\sqrt{-1+4t-4t^2+\mathrm{Csc[theta]}^2}\right)}{s\,\mathrm{Abs}\left[8(-1+t)t+2\,\mathrm{Cot[theta]}\left(-\mathrm{Cot[theta]}+\sqrt{-1+4t-4t^2+\mathrm{Csc[theta]}^2}\right)\right]}$

and as it turns out, the Abs part in the denominator is *always* negative. so remove abs and add a sign before integrating:

In[ ]:= FullSimplify[$\frac{2\,\mathrm{Csc[theta]}\left(\mathrm{Cot[theta]}-\sqrt{-1+4t-4t^2+\mathrm{Csc[theta]}^2}\right)}{s\left(8(-1+t)t+2\,\mathrm{Cot[theta]}\left(-\mathrm{Cot[theta]}+\sqrt{-1+4t-4t^2+\mathrm{Csc[theta]}^2}\right)\right)}$]

Out[ ]= $\frac{\mathrm{Csc[theta]}\left(\mathrm{Cot[theta]}-\sqrt{-(1-2t)^2+\mathrm{Csc[theta]}^2}\right)}{s\left(4(-1+t)t+\mathrm{Cot[theta]}\left(-\mathrm{Cot[theta]}+\sqrt{-(1-2t)^2+\mathrm{Csc[theta]}^2}\right)\right)}$

Sampling



- Sample $\theta \sim f_s(\theta) \cdot \sin\theta$
- Sample t

$$t = P^{-1}(\xi|\theta) = \boxed{\cos(\theta - \xi\theta)\sin(\xi\theta)/\sin\theta}$$

- Sample $\phi \sim \frac{1}{2\pi}$ (trivial)

PDF and Estimator

$$p(\boldsymbol{x}_1) = f_s(\theta)\,\frac{s}{d_1^2 \cdot d_2^2}\,\frac{\sin\theta}{\theta}$$

$$\hat{I} = f_c(\boldsymbol{x}_0 \leftrightarrow \boldsymbol{x}_2) \cdot \frac{\theta}{s\sin\theta}$$

PDF and Estimator

$$p(\boldsymbol{x}_1) = f_s(\theta) \frac{s}{d_1^2 \cdot d_2^2} \frac{\sin\theta}{\theta}$$

$$\hat{I} = f_c(\boldsymbol{x}_0 \leftrightarrow \boldsymbol{x}_2) \cdot \frac{\theta}{s \sin\theta}$$

- Phase function sampling $f_s \cdot \sin\theta$
  - Stock sampling routine: in solid angle, so theta slice contains Jacobian ($\sin\theta$)
  - Results in $\theta / \sin\theta$ in estimator (close to 1.0 for forward scattering)

PDF and Estimator

$$p(\boldsymbol{x}_1) = f_s(\theta) \frac{s}{d_1^2 \cdot d_2^2} \frac{\sin\theta}{\theta}$$

$$\hat{I} = f_c(\boldsymbol{x}_0 \leftrightarrow \boldsymbol{x}_2) \cdot \frac{\theta}{s \sin\theta}$$

- PDF contains geometry terms: $d_1^2 \cdot d_2^2$:)
- Also contains s, the distance $\boldsymbol{x}_0$ to $\boldsymbol{x}_2$
  - Similar to Kalos 1963/equiangular sampling: replaced $1/d^2$ singularity by $1/s$
  - We include the (important!) phase function
  - Our distance s is between $\boldsymbol{x}_0, \boldsymbol{x}_2$ (not $\boldsymbol{x}_1$)

PDF and Estimator

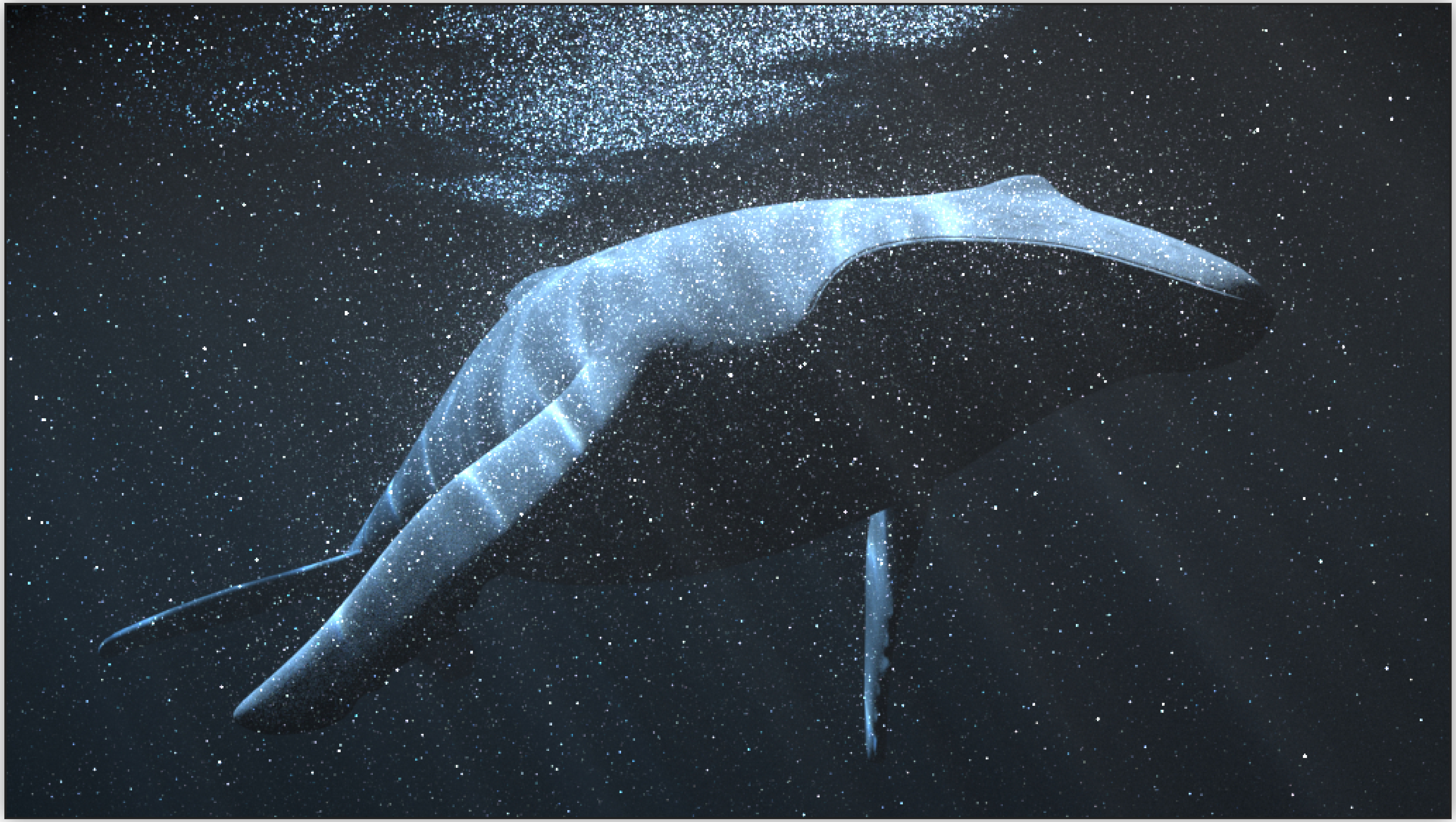
$$p(\boldsymbol{x}_1) = f_s(\theta)\,\frac{s}{d_1^2 \cdot d_2^2}\,\frac{\sin\theta}{\theta}$$

$$\hat{I} = f_c(\boldsymbol{x}_0 \leftrightarrow \boldsymbol{x}_2) \cdot \frac{\theta}{s\sin\theta}$$

- It remains to mention the unsampled f_c

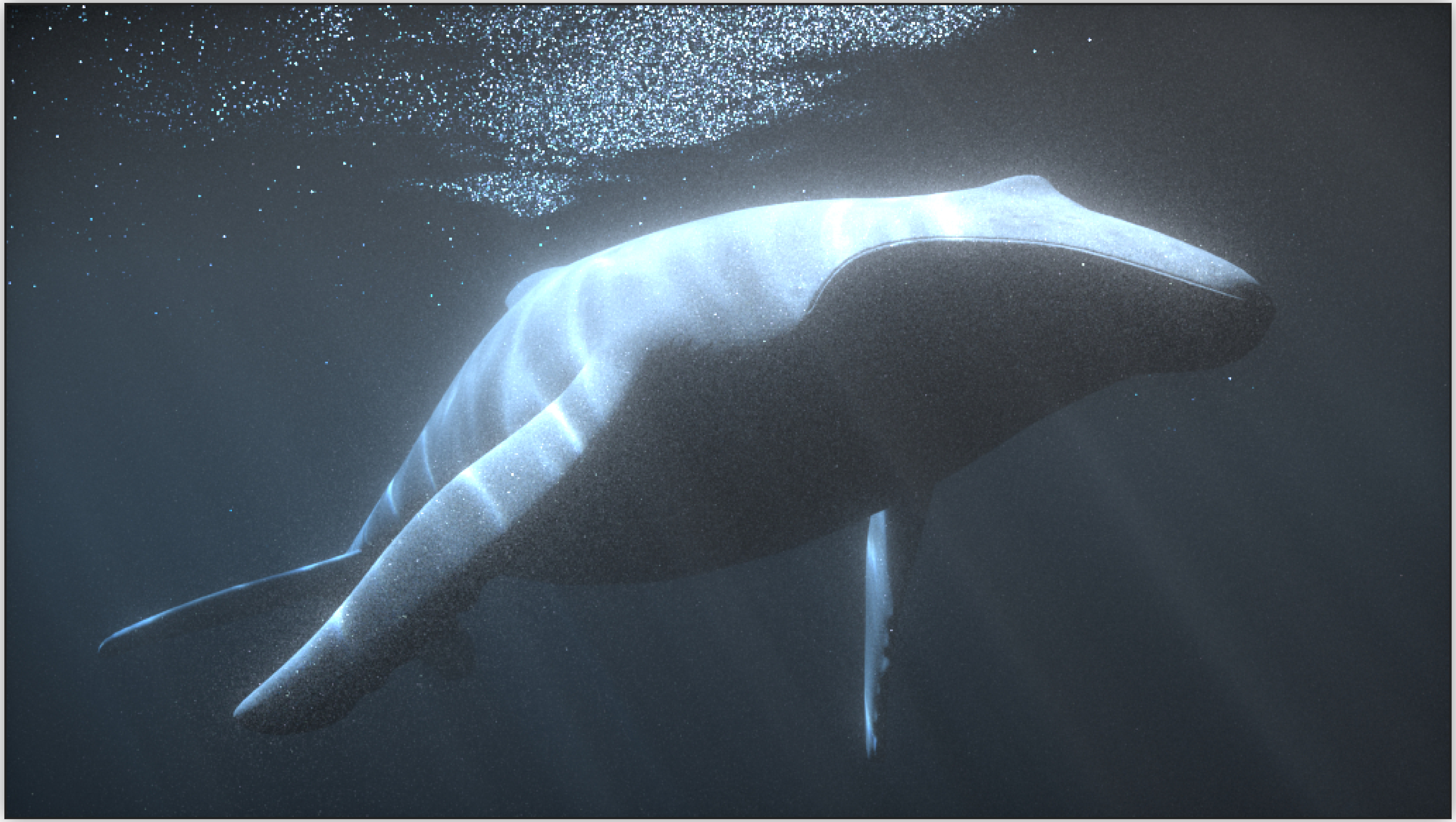
$$f_c(\boldsymbol{x}_0 \leftrightarrow \boldsymbol{x}_2) = \cos\theta_0 \cdot \cos\theta_2 \cdot \mu_s \cdot T(\boldsymbol{x}_0, \boldsymbol{x}_1) \cdot T(\boldsymbol{x}_1, \boldsymbol{x}_2) \cdot W(\boldsymbol{x}_2)$$

Results



equiangular sampling

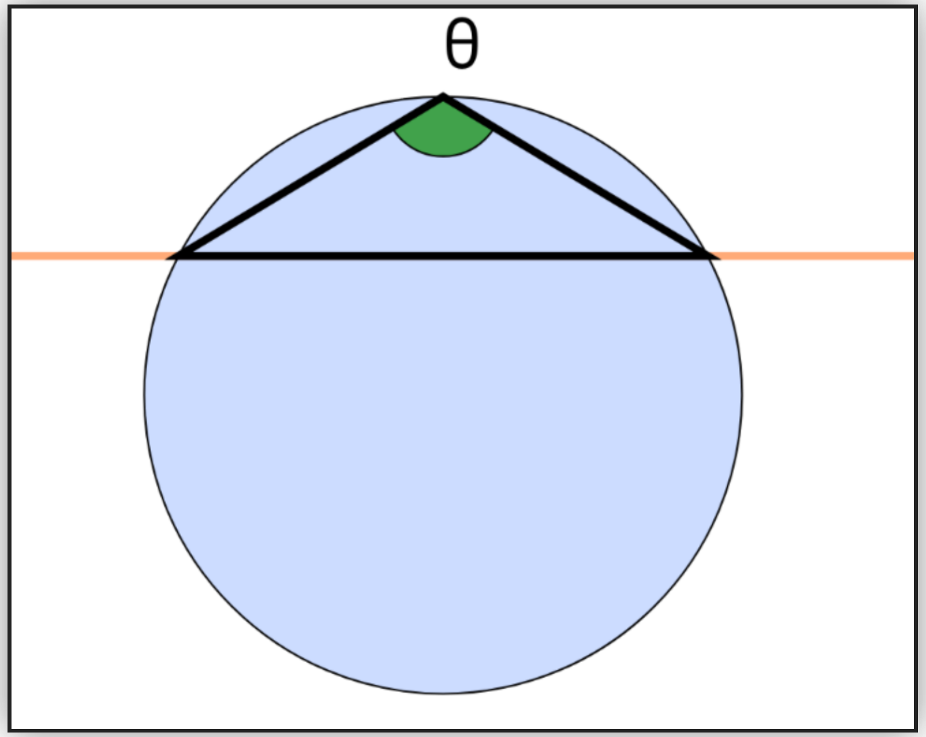
Results



OMNEE (ours)

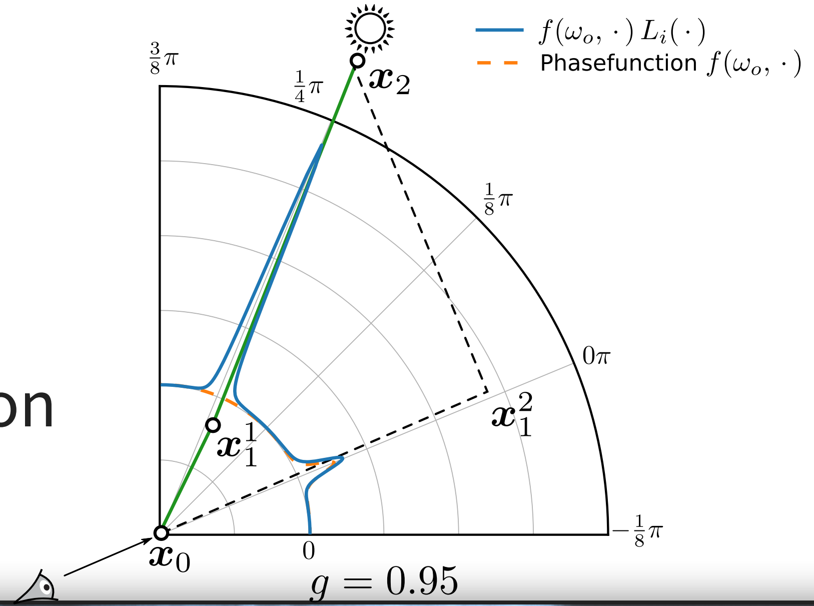
Conclusion

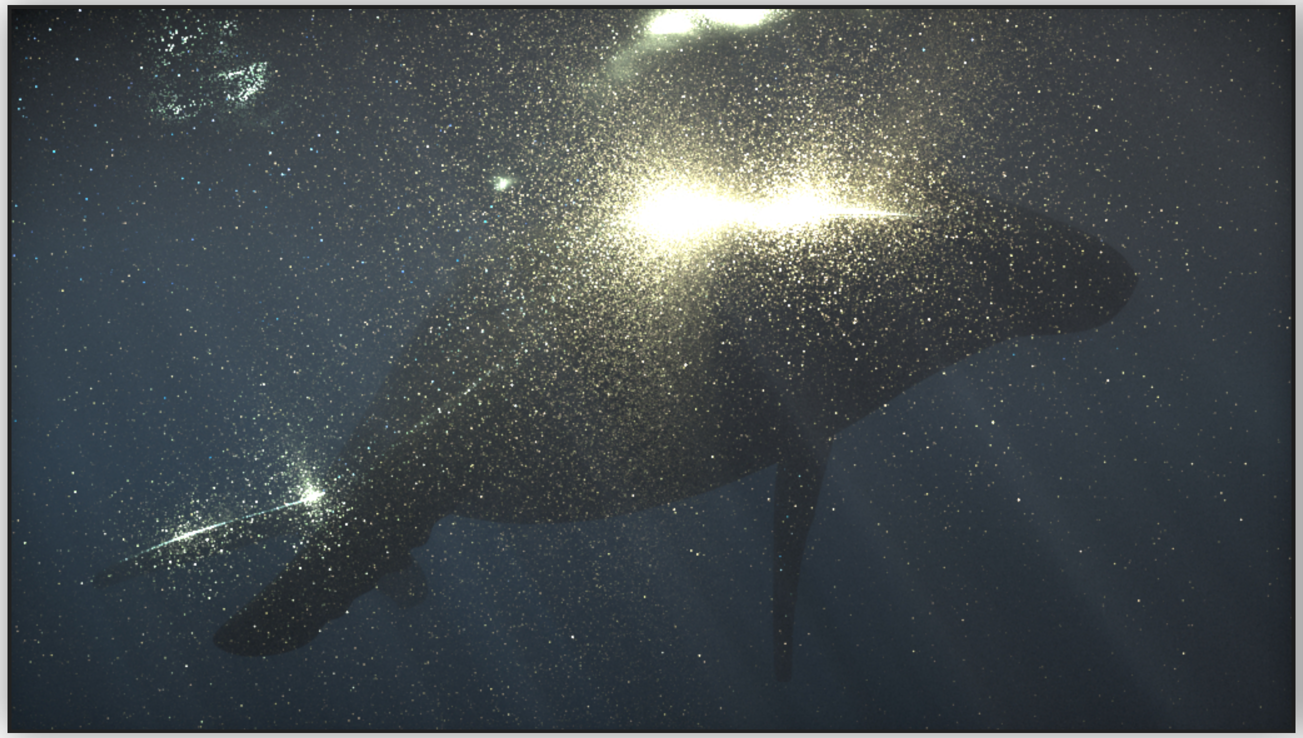
- Hope that we showed:
  - Highly peaked forward scattering phase functions are important for the look!
  - No efficient technique available to us previously
- We provide *once-more collided flux*:
  - Sampling the angle at the extra vertex first!
  - Core of the technique: triangular geometry with circumcircle

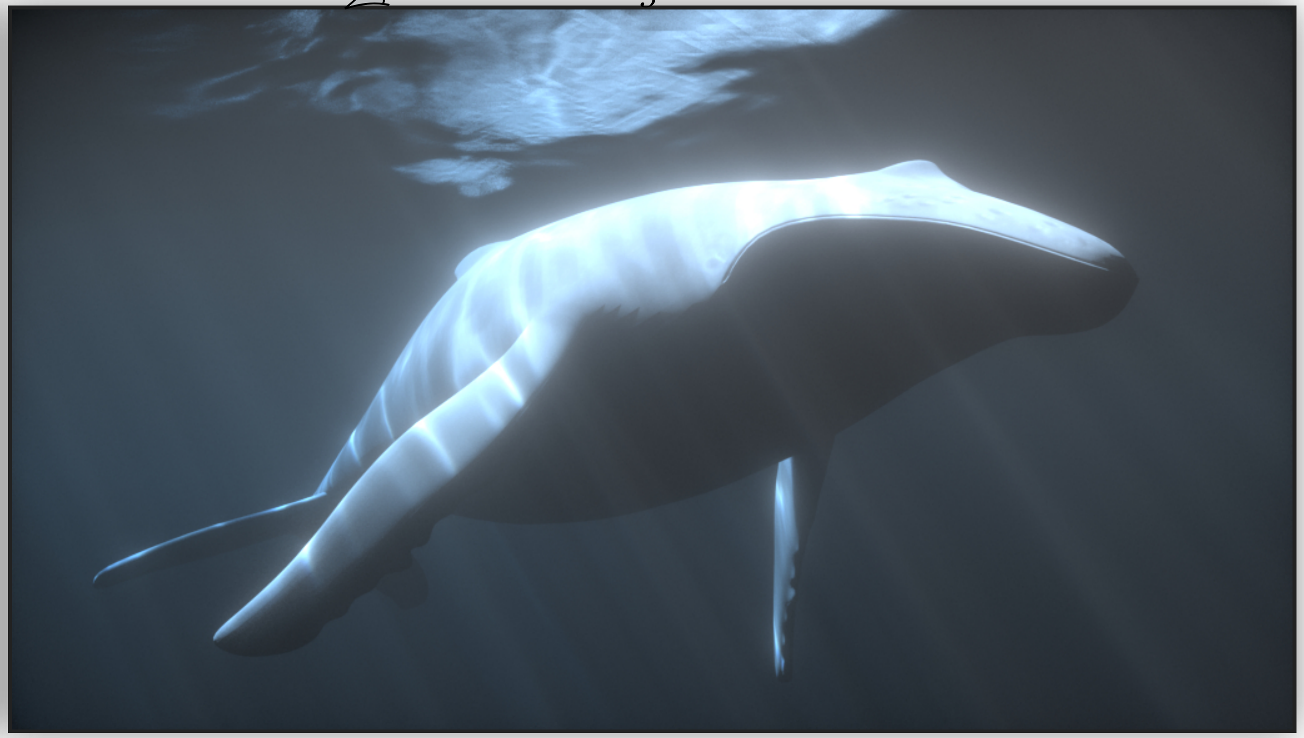


Discussion/Limitations

- Forward scattering only!
- We don't sample transmittance
  - But does it matter? Can be sampled with phase function
- We don't sample the BSDF/phase function at $\boldsymbol{x}_0$:

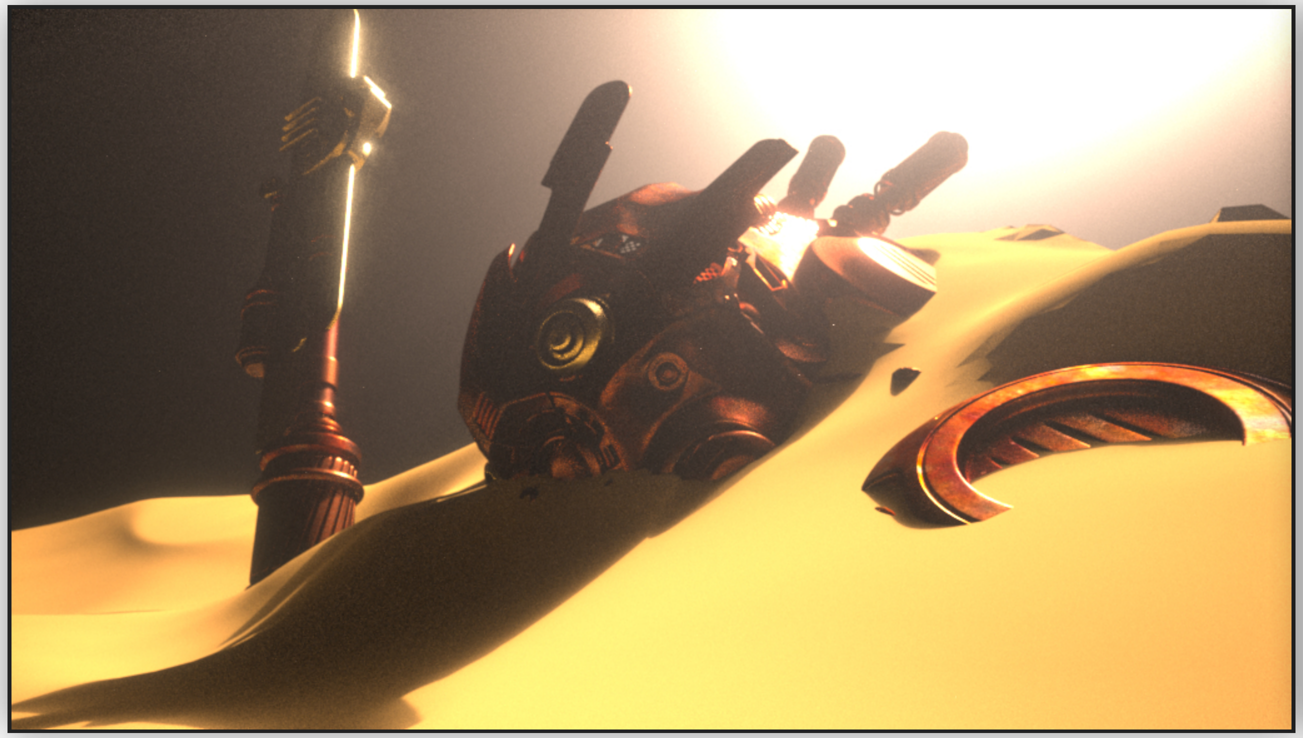


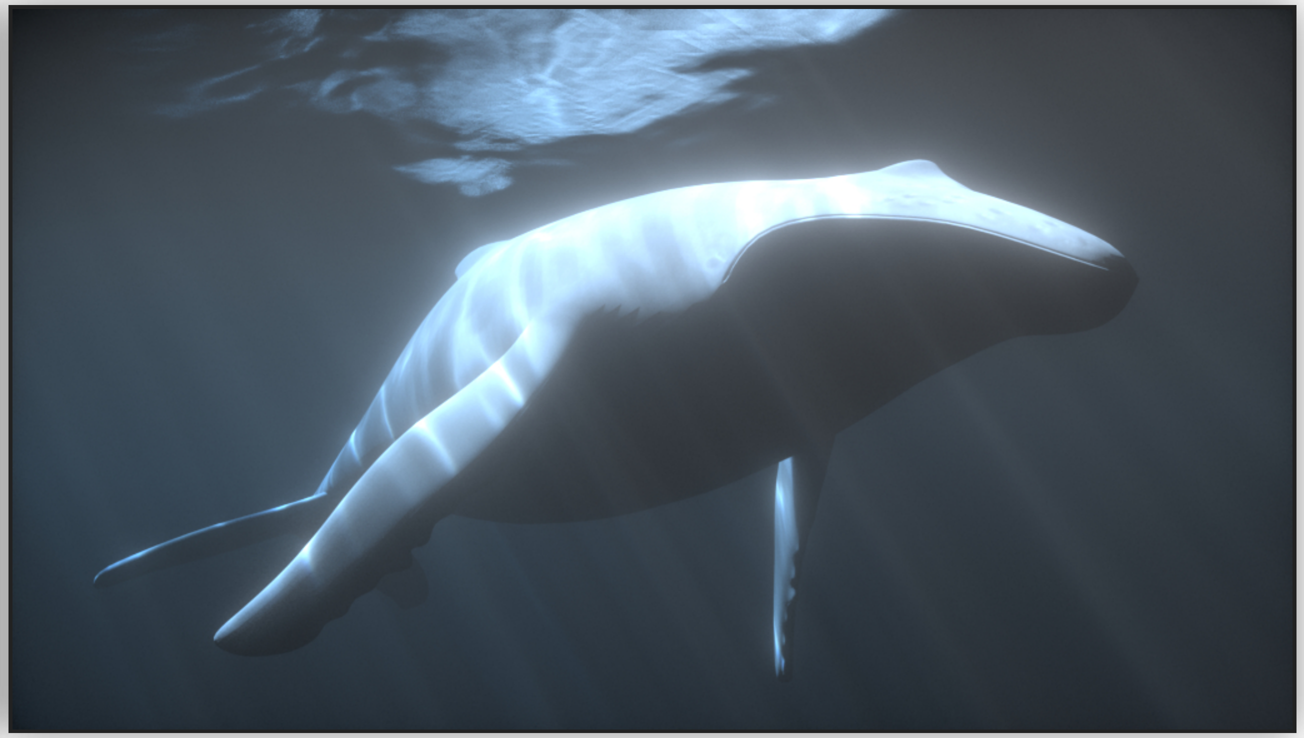




- Can this lead to a parameterisation for more generic multiple scattering?

Thank you for listening!

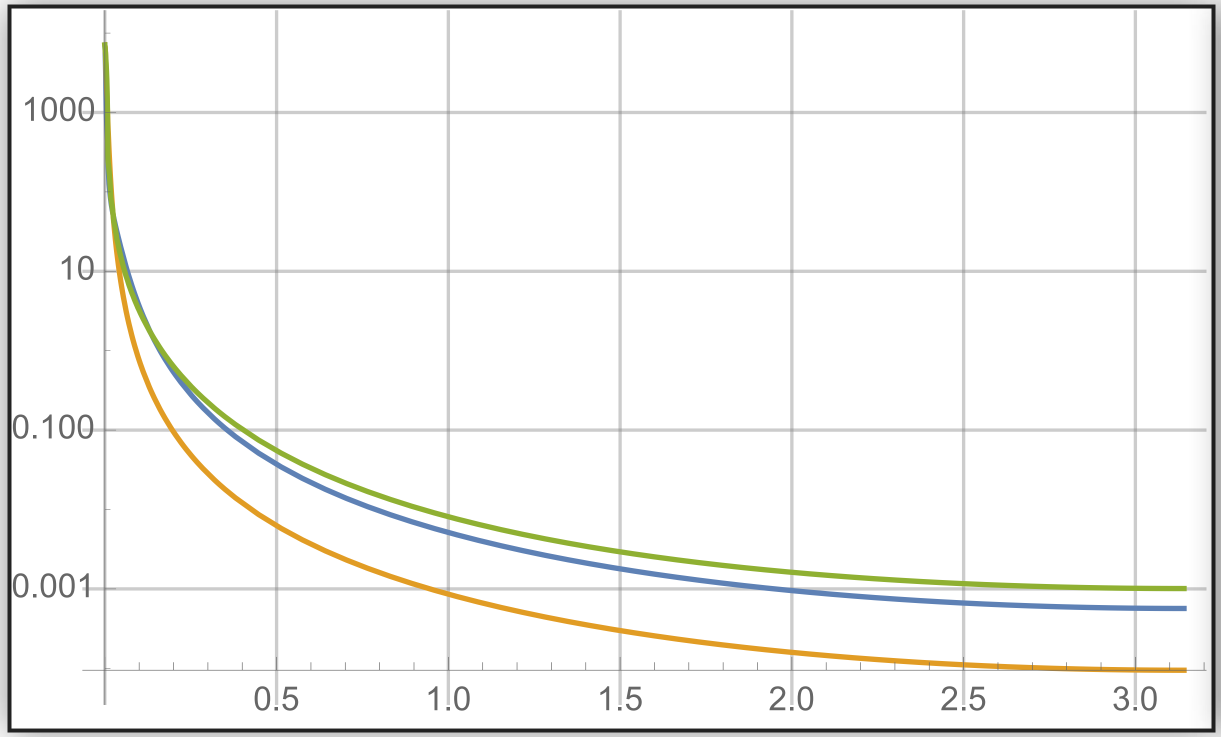


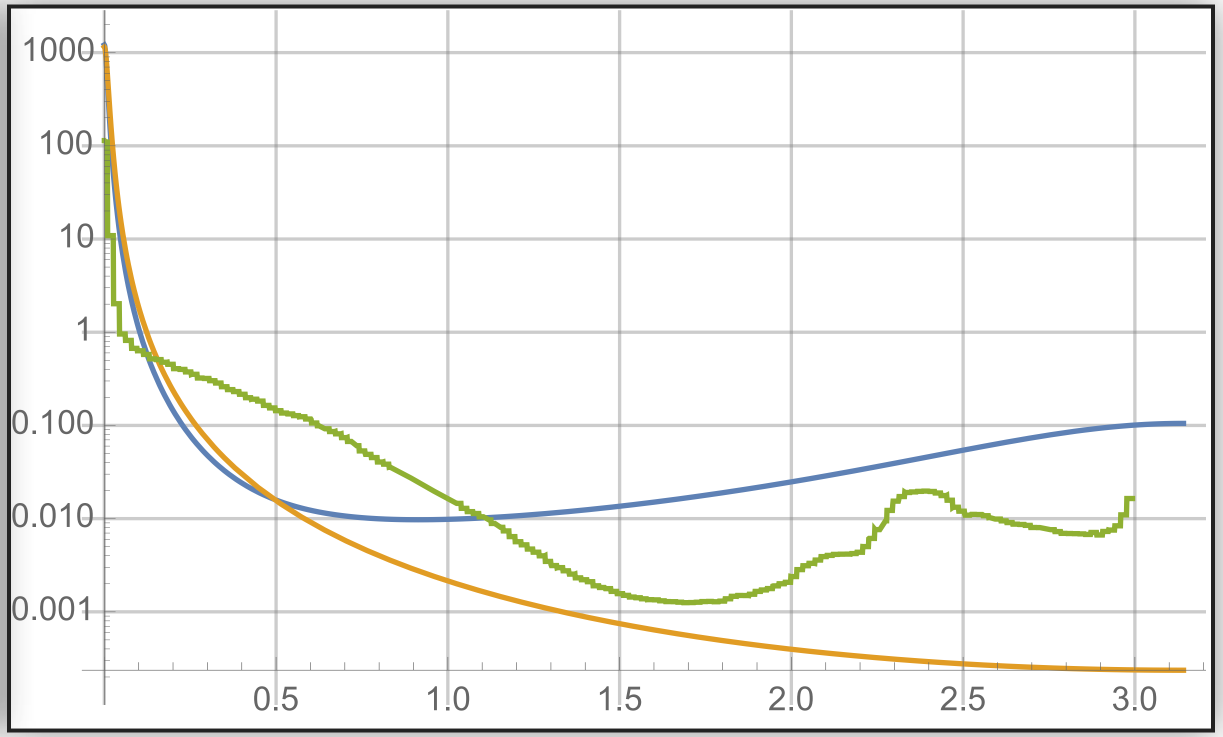


Backup slides

Typical phase functions

- Are super peaky forward scattering (for water/haze):
- Two term HG fit: $g_1 = 0.997, g_2 = 0.960$ and $g_1 = 0.990, g_2 = -0.440$





- Left: Fournier/Forand, Right: Mie
- But is that visually important? What does not-so-peaky look like?