Microfacet-Based Normal Mapping
for Robust Monte Carlo Path Tracing

Analytic Diffuse Multiple-Scattering

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In Section 6.1 of our paper we explain how we can compute the multiple-scattering BRDF analytically if we use a diffuse micro-BRDF for both facets. We derive this analytic diffuse multiple-scattering BRDF using a radiosity method.

The initial radiosity, i.e. the direct illumination, of the microfacets with normal $\omega_m$ per irradiance from direction $\omega_o$ is

$$b^f(\omega_o, \omega_m) = \rho \langle \omega_o \cdot \omega_m \rangle G_1(\omega_o, \omega_m)$$

By inversion of the $2 \times 2$ transport matrix we obtain the radiosity of each facet after an infinite number of reflections as a function of their initial radiosities:

$$B_{\omega_p}(\omega_o) = \frac{b^f(\omega_o, \omega_p) + \rho F_{p-t} b^f(\omega_o, \omega_t)}{1 - \rho^2 F_{p-t} F_{t-p}}$$

$$B_{\omega_t}(\omega_o) = \frac{b^f(\omega_o, \omega_p) \rho F_{t-p} + b^f(\omega_o, \omega_t)}{1 - \rho^2 F_{p-t} F_{t-p}},$$

where $F_{p-t}$ and $F_{t-p}$ are given by the equation for the “form factor between two infinitely long plates of unequal width $(a, b)$ having a common edge with an included angle $\alpha$” from Howell [2010]:

$$F_{p-t} = \frac{\sqrt{1 - (\omega_m \cdot \omega_g)^2} + 1 - (\omega_m \cdot \omega_g)}{2}$$

$$F_{t-p} = \frac{1}{2} + \frac{1 - (\omega_m \cdot \omega_g)}{2 \sqrt{1 - (\omega_m \cdot \omega_g)^2}}.$$

Finally, the multiple-scattering BRDF is the radiosity of the two facets weighted by their intersection probabilities for the incident direction:

$$f_\infty(\omega_i, \omega_o) \langle \omega_o, \omega_g \rangle = \frac{1}{\pi} [\lambda_p(\omega_i) B_{\omega_p}(\omega_o) + \lambda_t(\omega_i) B_{\omega_t}(\omega_o)].$$
References