Microfacet-Based Normal Mapping for Robust Monte Carlo Path Tracing

Analytic Diffuse Multiple-Scattering

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In Section 6.1 of our paper we explain how we can compute the multiple-scattering BRDF analytically if we use a diffuse micro-BRDF for both facets. We derive this analytic diffuse multiple-scattering BRDF using a radiosity method.

The initial radiosity, i.e. the direct illumination, of the microfacets with normal ω_m per irradiance from direction ω_o is

$$b^e(\omega_o, \omega_m) = \rho \, \langle \omega_o \cdot \omega_m \rangle \, G_1(\omega_o, \omega_m) \tag{1}$$

By inversion of the 2×2 transport matrix we obtain the radiosity of each facet after an infinite number of reflections as a function of their initial radiosities:

$$\begin{aligned} B_{\omega_p}(\omega_o) &= \frac{b^e(\omega_o, \omega_p) + \rho \, F_{p-t} b^e(\omega_o, \omega_t)}{1 - \rho^2 \, F_{p-t} \, F_{t-p}} \\ B_{\omega_t}(\omega_o) &= \frac{b^e(\omega_o, \omega_p) \rho \, F_{t-p} + b^e(\omega_o, \omega_t)}{1 - \rho^2 \, F_{p-t} \, F_{t-p}}, \end{aligned} \tag{2}$$

where F_{p-t} and F_{t-p} are given by the equation for the "form factor between two infinitely long plates of unequal width (a, b) having a common edge with an included angle α" from Howell [2010]:

$$F_{p-t} = \frac{\sqrt{1 - (\omega_m \cdot \omega_g)^2} + 1 - (\omega_m \cdot \omega_g)}{2}, \tag{3}$$

$$F_{t-p} = \frac{1}{2} + \frac{1 - (\omega_m \cdot \omega_g)}{2\sqrt{1 - (\omega_m \cdot \omega_g)^2}}. \tag{4}$$

Finally, the multiple-scattering BRDF is the radiosity of the two facets weighted by their intersection probabilities for the incident direction:

$$f_\infty(\omega_i, \omega_o) \, \langle \omega_o, \omega_g \rangle = \frac{1}{\pi} \left[\lambda_p(\omega_i) \, B_{\omega_p}(\omega_o) + \lambda_t(\omega_i) \, B_{\omega_t}(\omega_o) \right]. \tag{5}$$

References

John R. Howell. 2010. A catalog of radiation heat transfer configuration factors. (2010). http://www.thermalradiation.net/indexCat.html.