Sparse high-degree polynomials for wide-angle lenses

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motivation

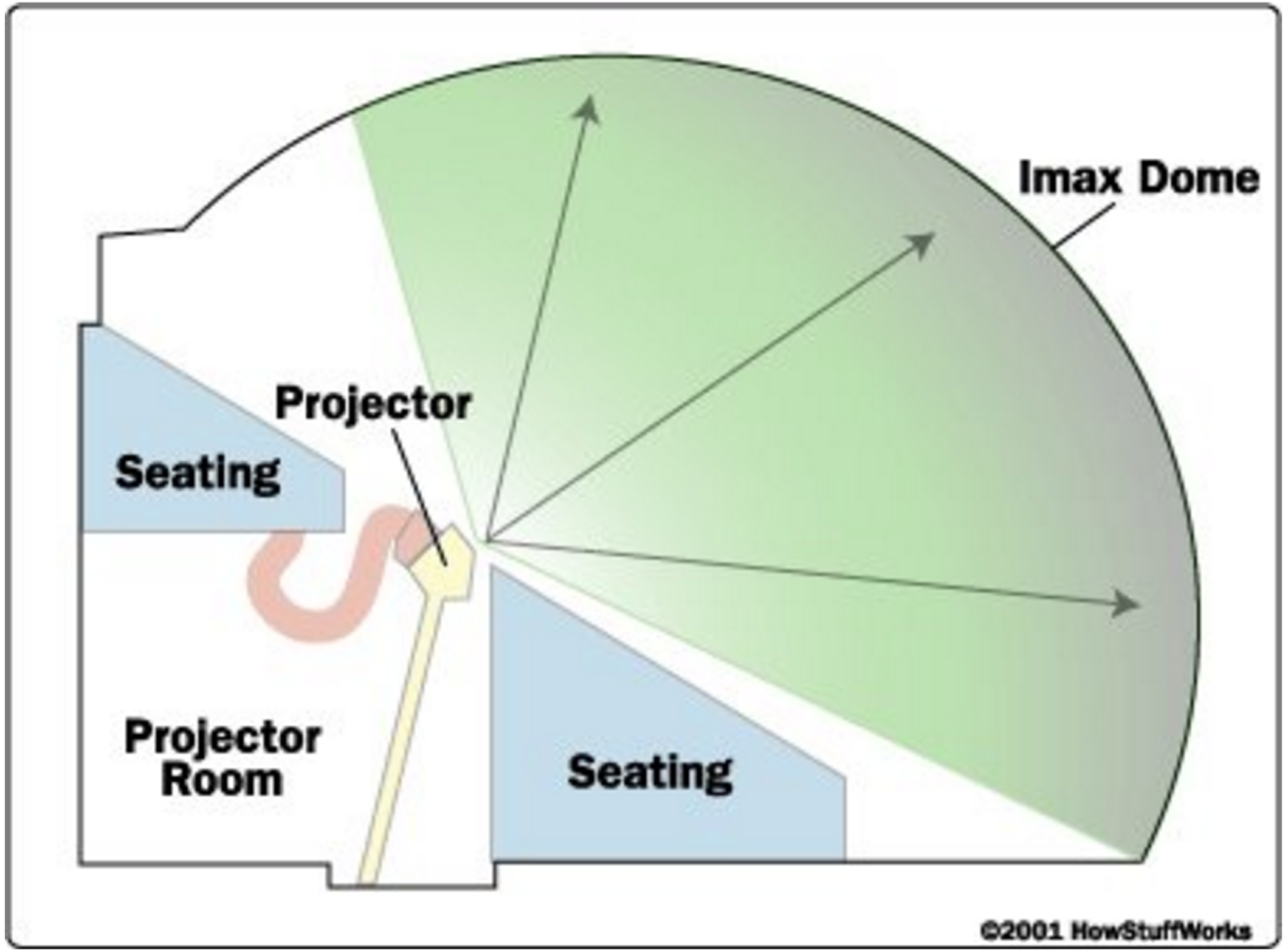
fisheye lenses

motivation

panoramic cameras

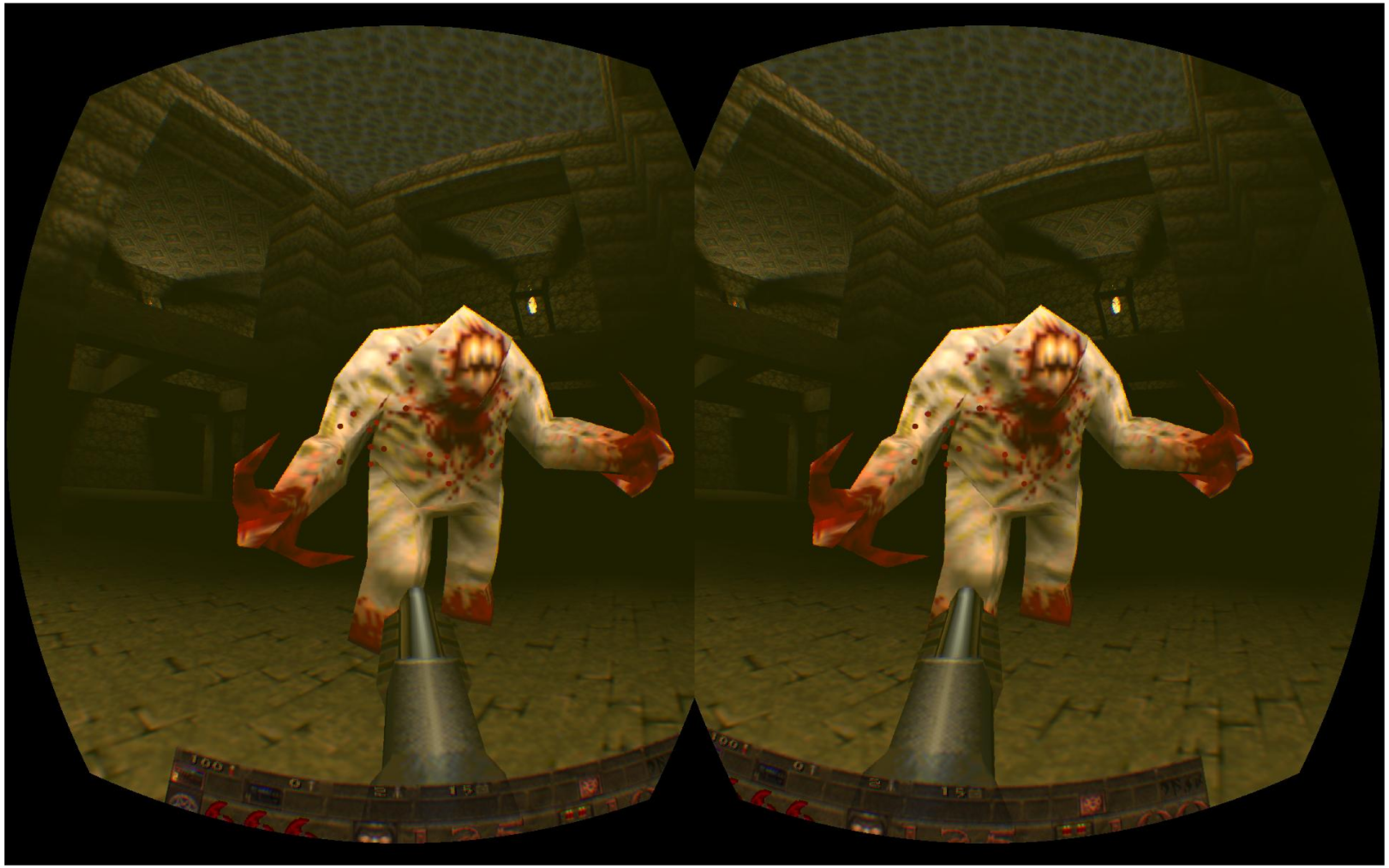
motivation

rendering for the imax dome



motivation

rendering for virtual reality



motivation

photorealistic rendering

- flat and boring bokeh

motivation

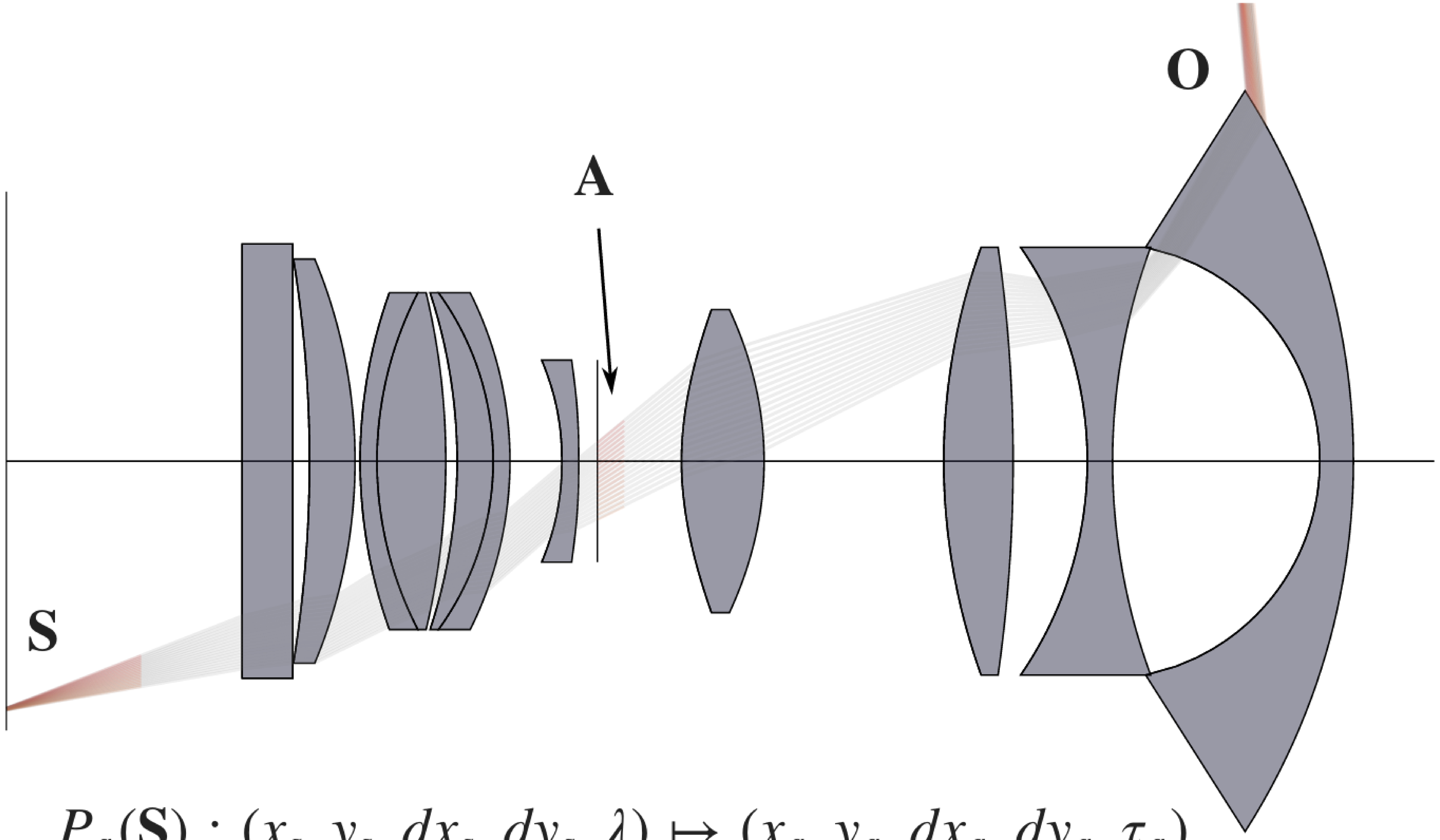
photorealistic rendering

- interesting bokeh, distortion, and vignetting to match plate

state of the art

approximate lens systems with simple polynomial

- collapse complicated ray tracing
- simple function evaluations $\mathbf{A} = P_a(\mathbf{S})$ and $\mathbf{O} = P_o(\mathbf{S})$

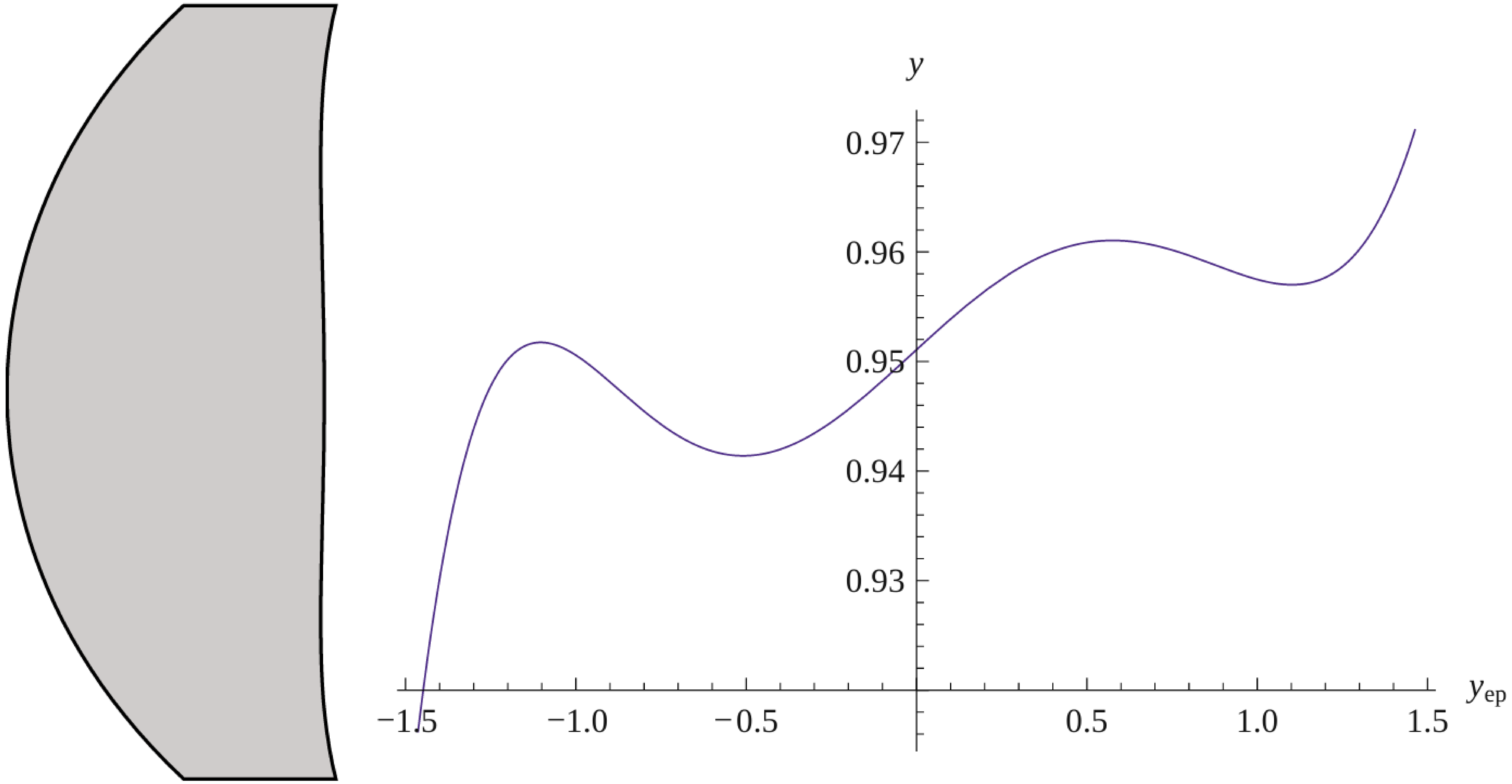


$$P_a(\mathbf{S}) : (x_s, y_s, dx_s, dy_s, \lambda) \mapsto (x_a, y_a, dx_a, dy_a, \tau_a)$$
$$P_o(\mathbf{S}) : (x_s, y_s, dx_s, dy_s, \lambda) \mapsto (x_o, y_o, dx_o, dy_o, \tau_o)$$

state of the art

optics use polynomials to ray trace

- approximate ray tracing using polynomials for lens designers [ZHB10]
- and analyse error in that domain directly

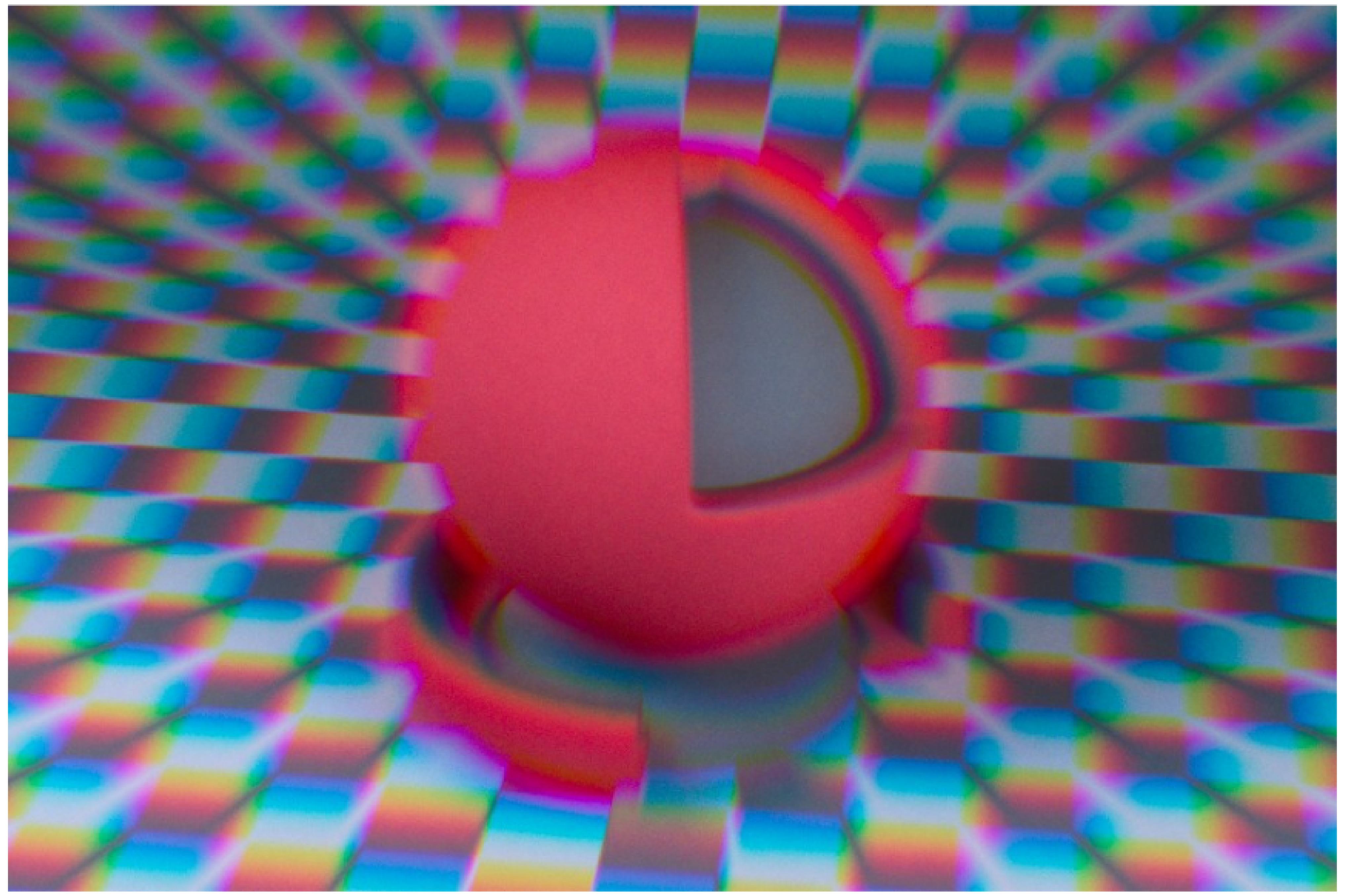


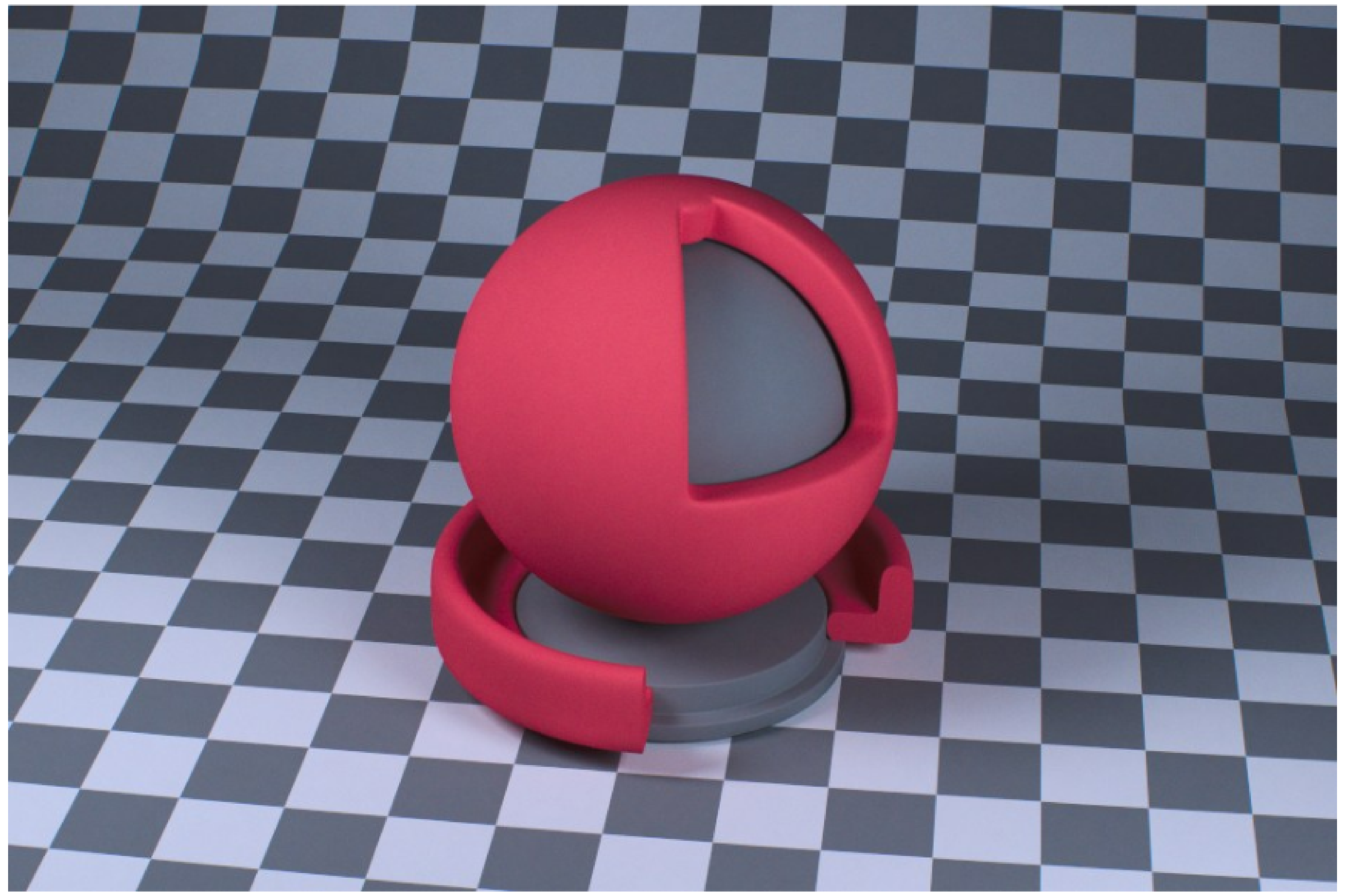
(a) (b)

state of the art

all based on Taylor expansion

- scary formulas for analytic differentiation required!
- not precise in outer rims [HD14]
    - use Taylor configuration, optimise coefficients





state of the art

Taylor polynomials

- don't like cumbersome analytic Taylor expansions
  - needs pen and paper or computer algebra software
  - expand every lens element, insert, re-truncate polynomials
- want polynomial with only few coefficients (fast evaluation!)
- and more precision (analytic Taylor expansion hardly tractable for high degree)
  - idea: select from higher-degree terms

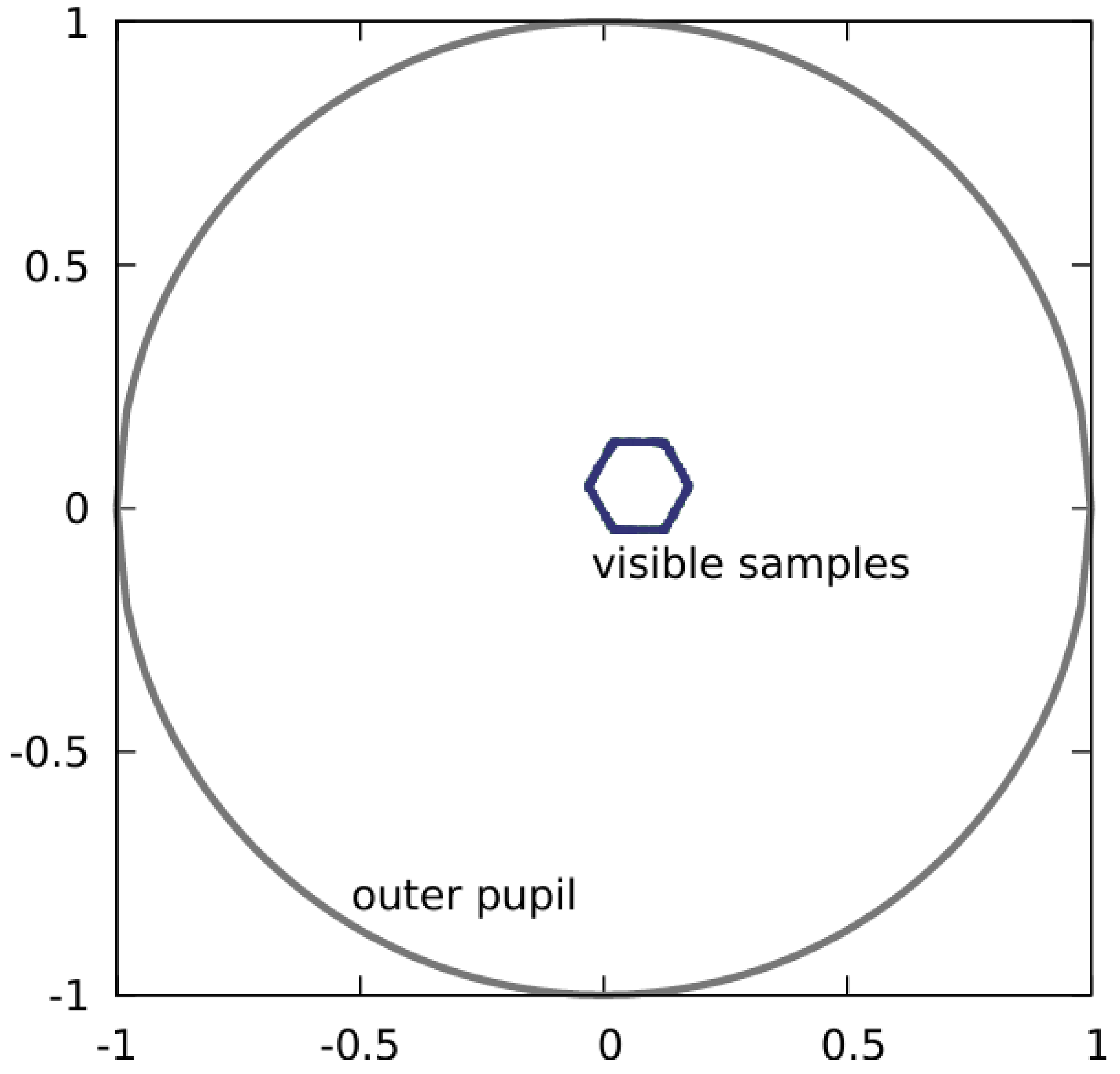
$$\cos x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

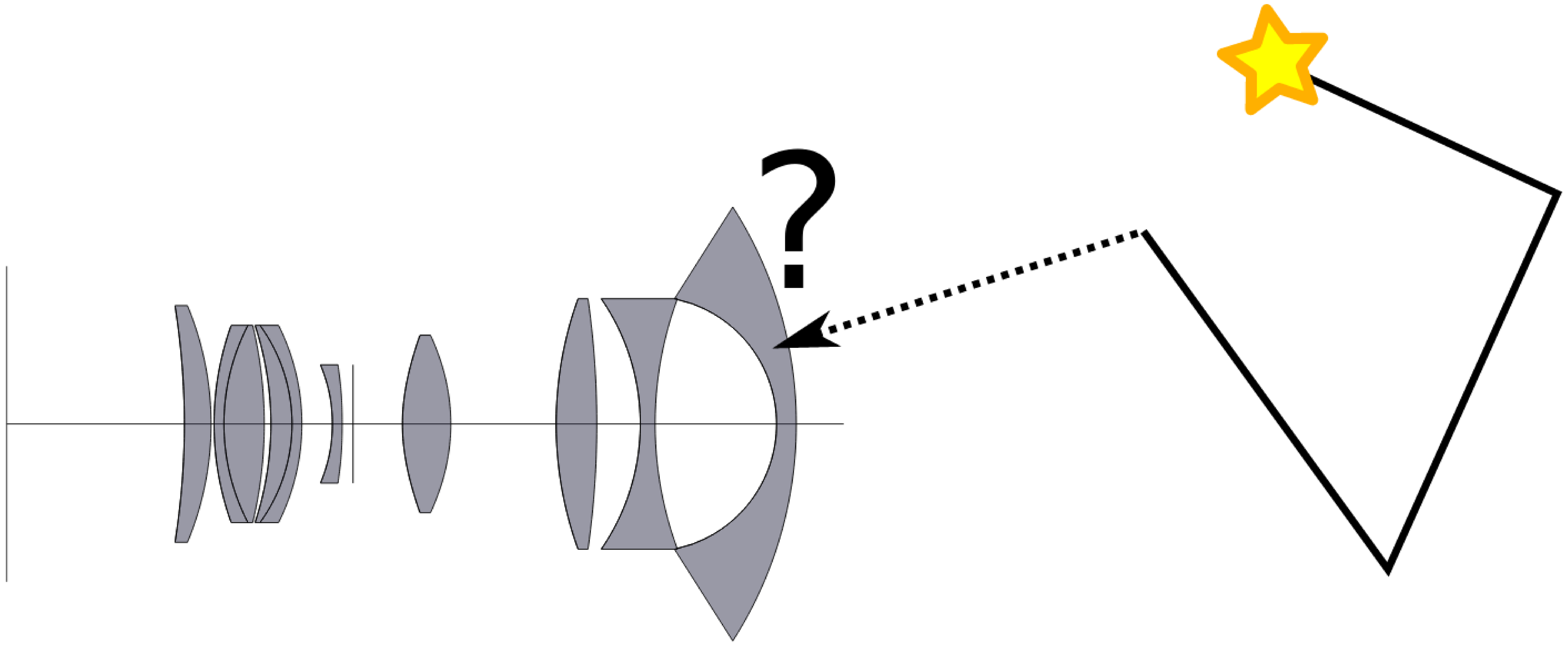
$$= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 + \cdots$$

state of the art

fisheye lenses

- precision in periphery *is* important!
- current lens connections (for light tracing)
  - sample outer pupil uniformly
  - have terrible performance



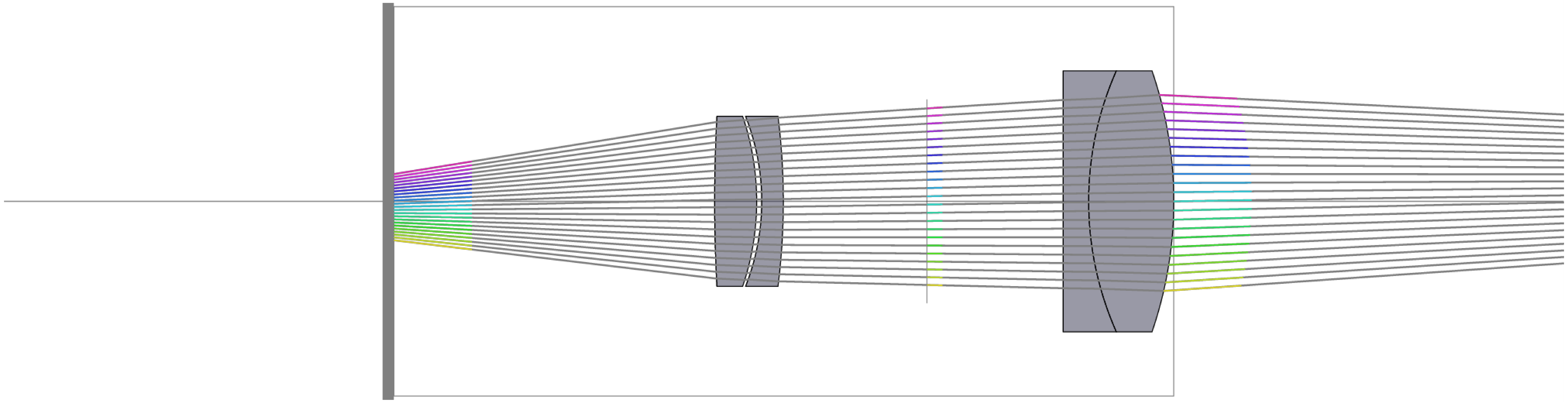


state of the art

sampling the outer pupil

- why didn't we bother earlier?
  - not such a bad strategy for long lenses

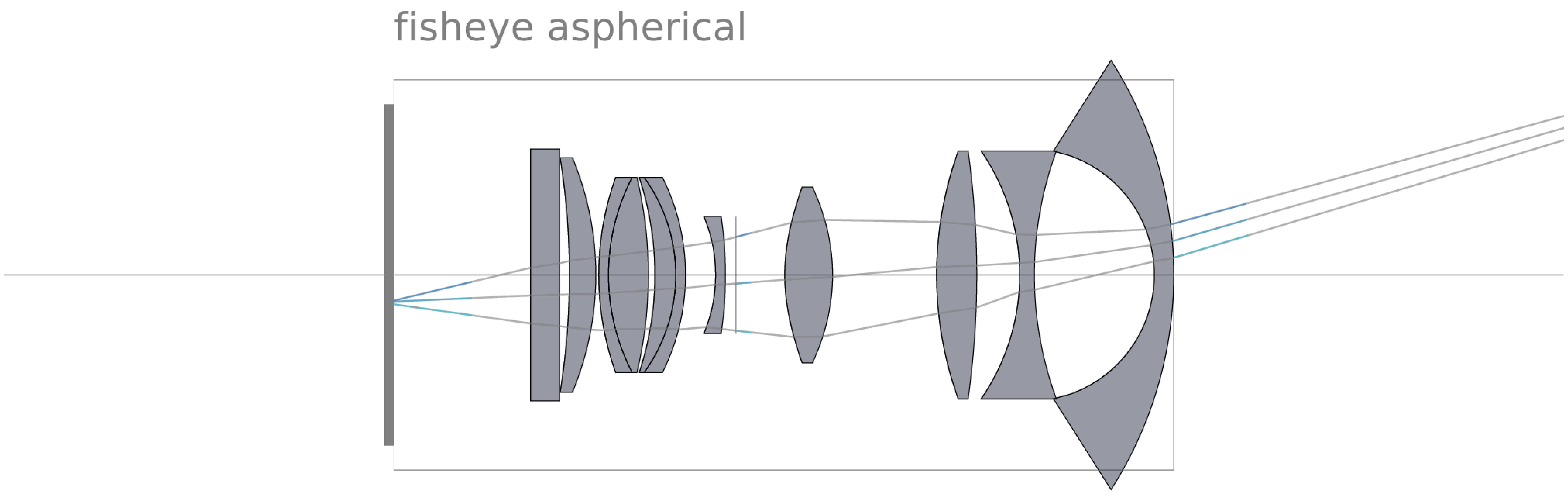
petzval kodak



state of the art

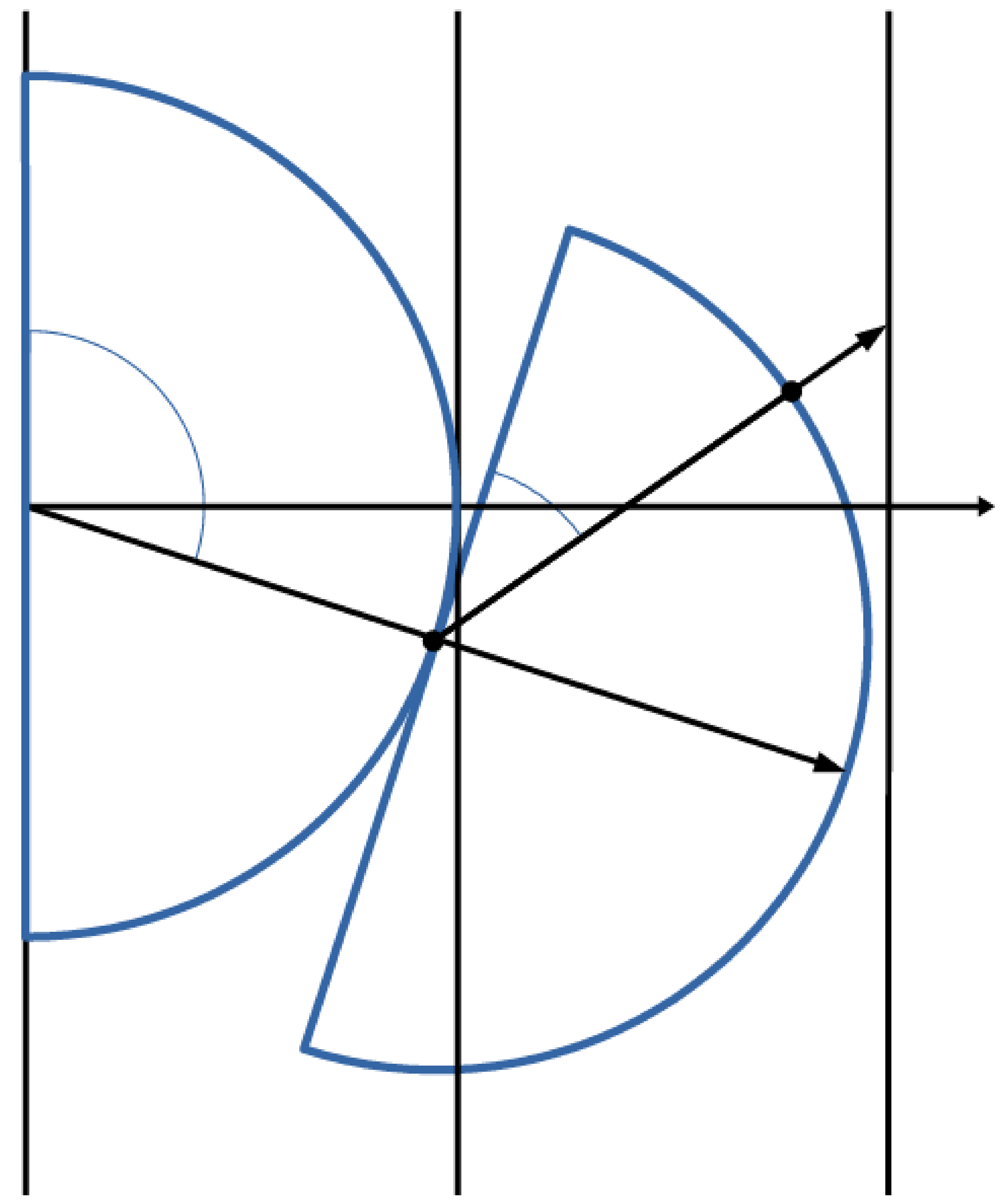
sampling the outer pupil

- why didn't we bother earlier?
  - but terrible for fisheyes



technical contributions

- parametrise the light fields for fisheyes
  - no plane/plane 180-degree limit



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  - use orthogonal matching pursuit (OMP)
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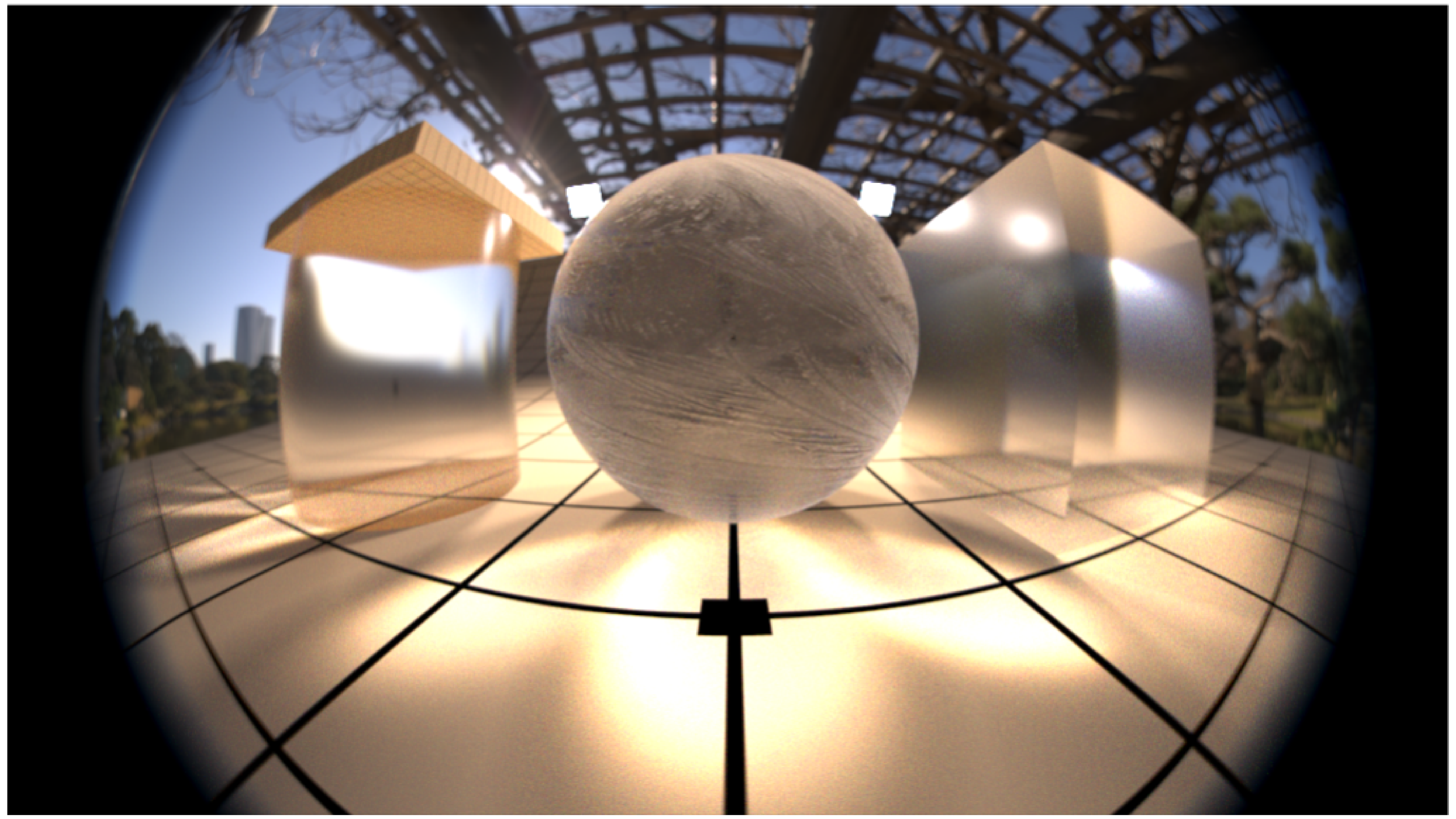

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- fast GPU preview rendering implementation

parametrisation suitable for fisheye lenses

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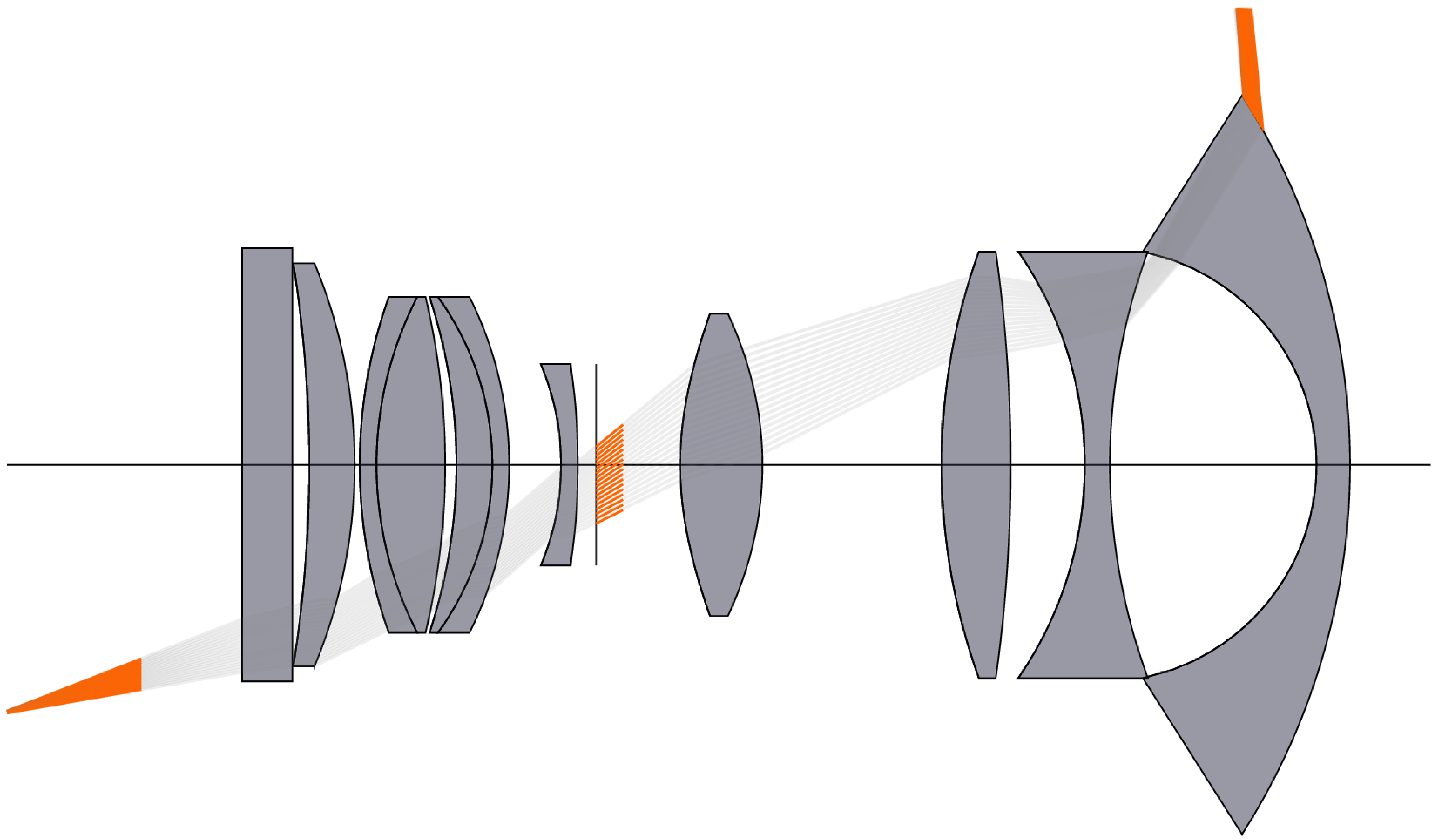
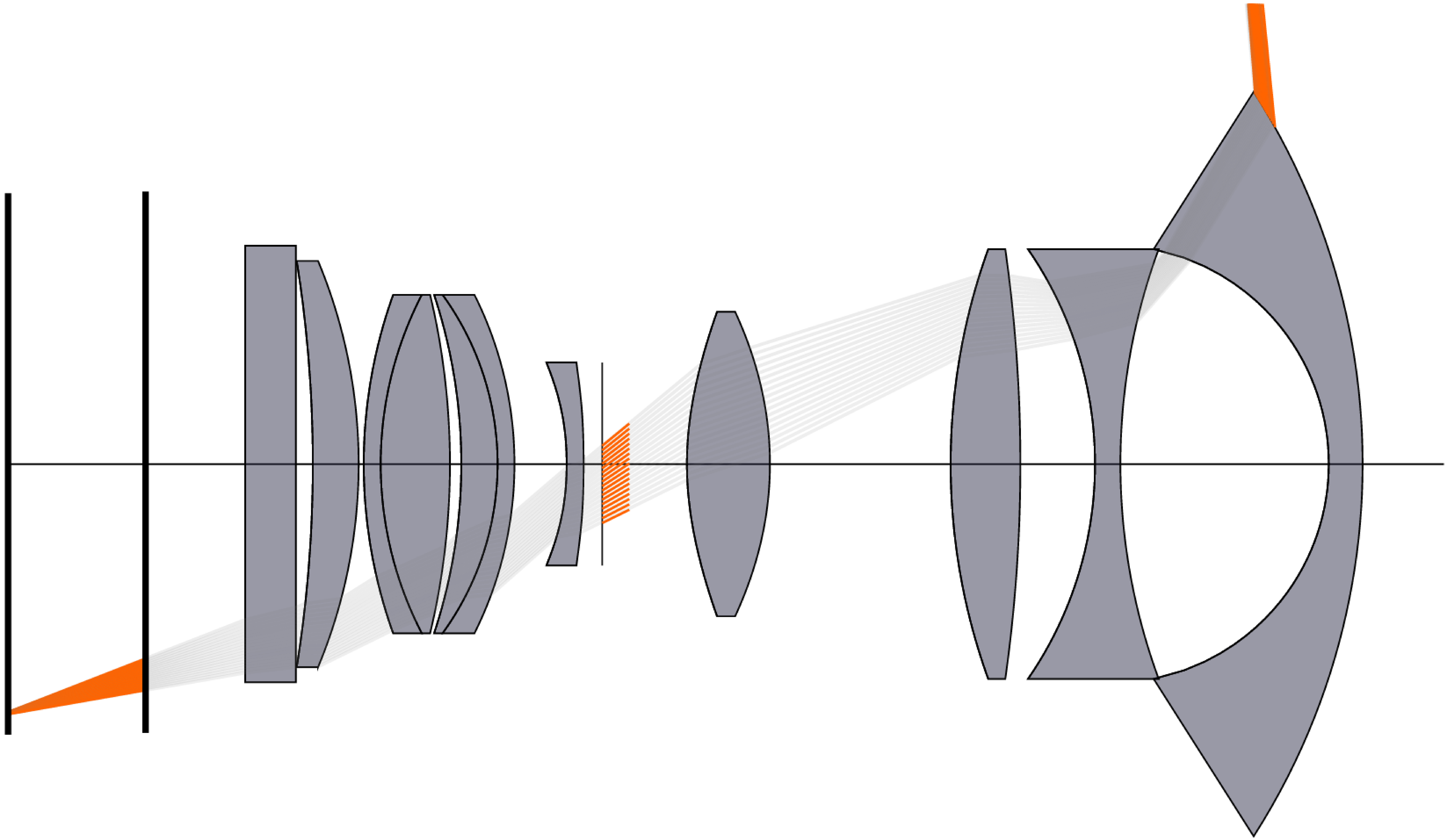


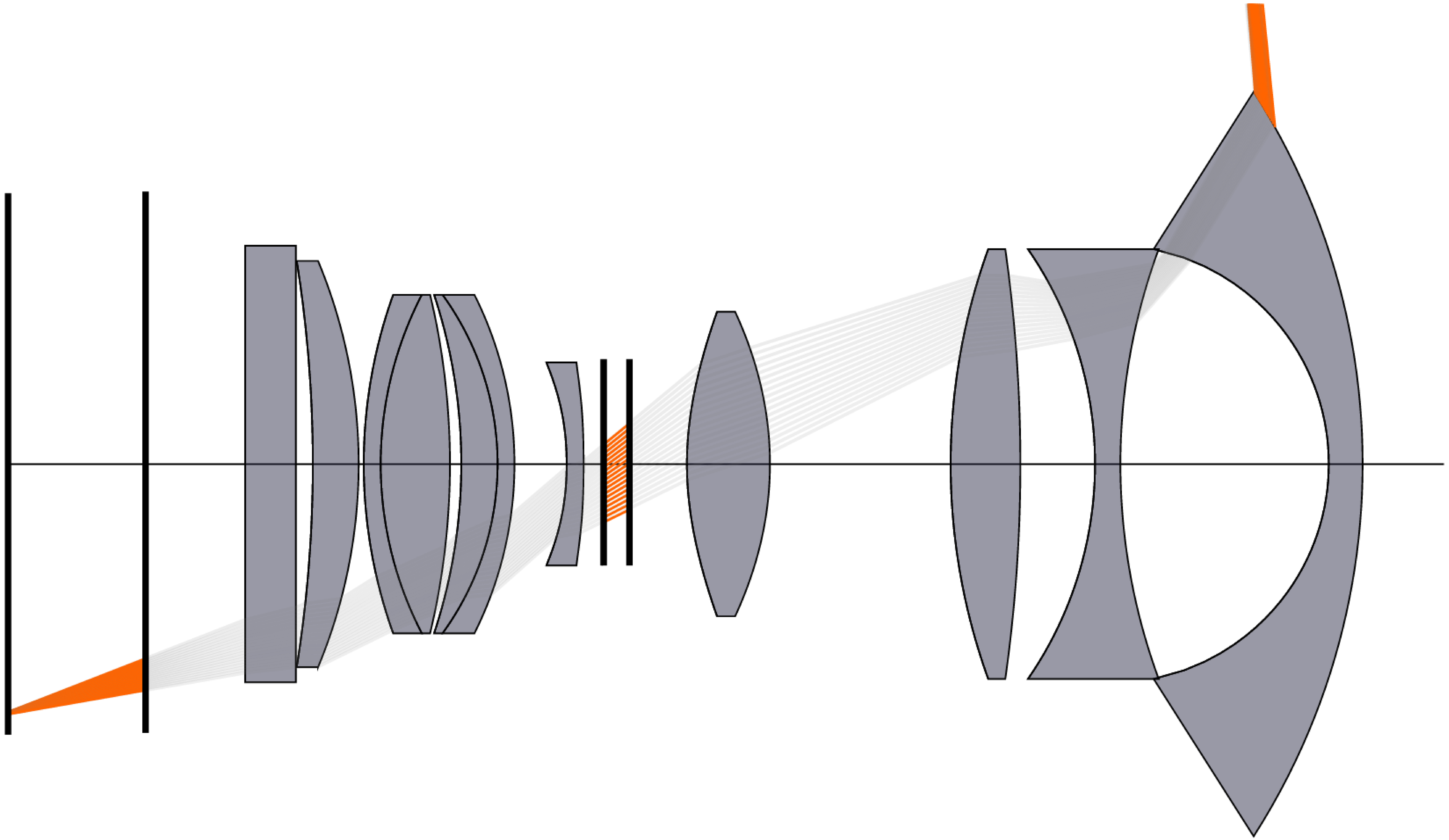
parametrisation suitable for fisheye lenses

- plane/plane on sensor



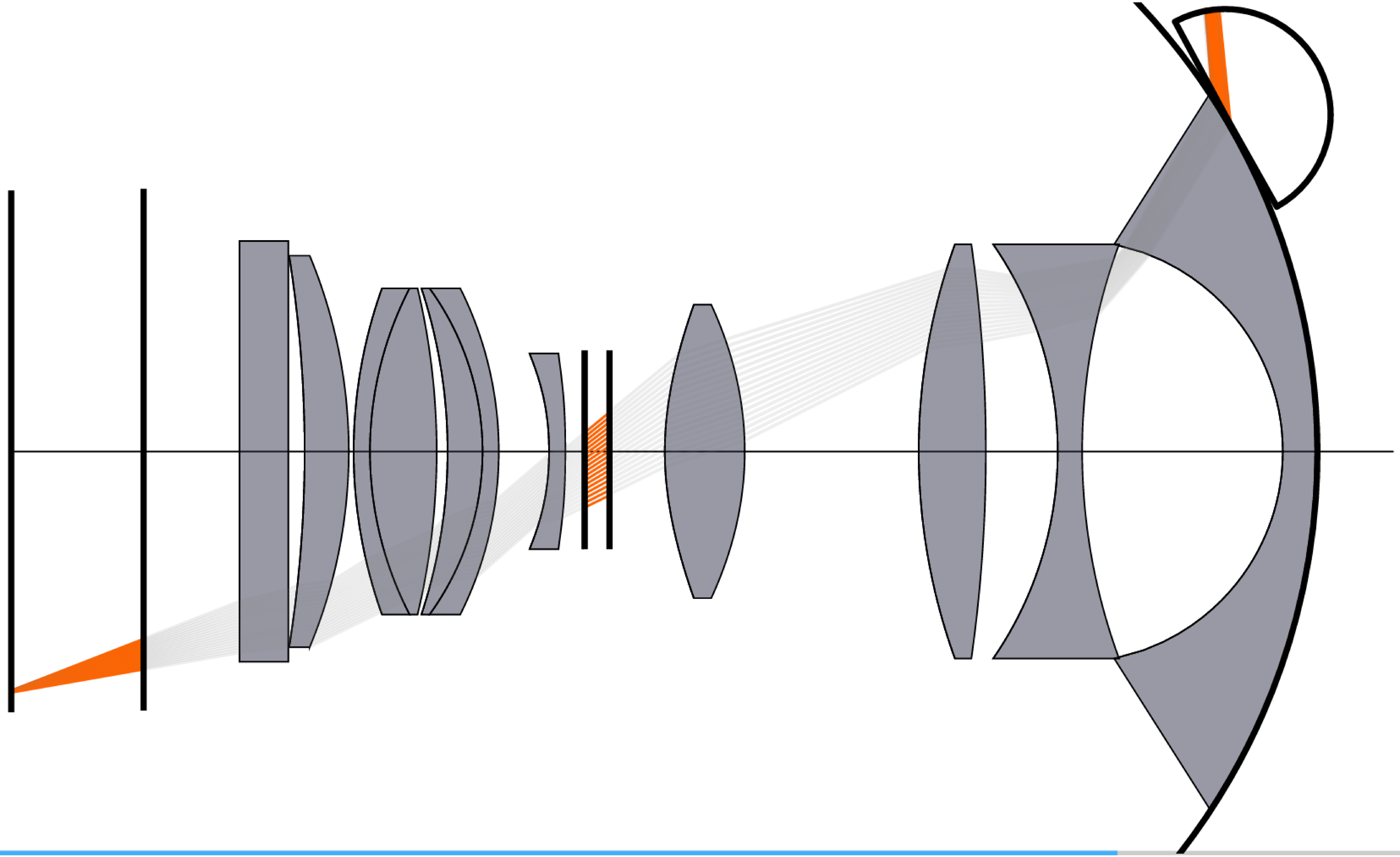
parametrisation suitable for fisheye lenses

- plane/plane on sensor
- plane/plane on aperture



parametrisation suitable for fisheye lenses

- plane/plane on sensor
- plane/plane on aperture
- hemi-sphere/hemi-sphere on outer pupil



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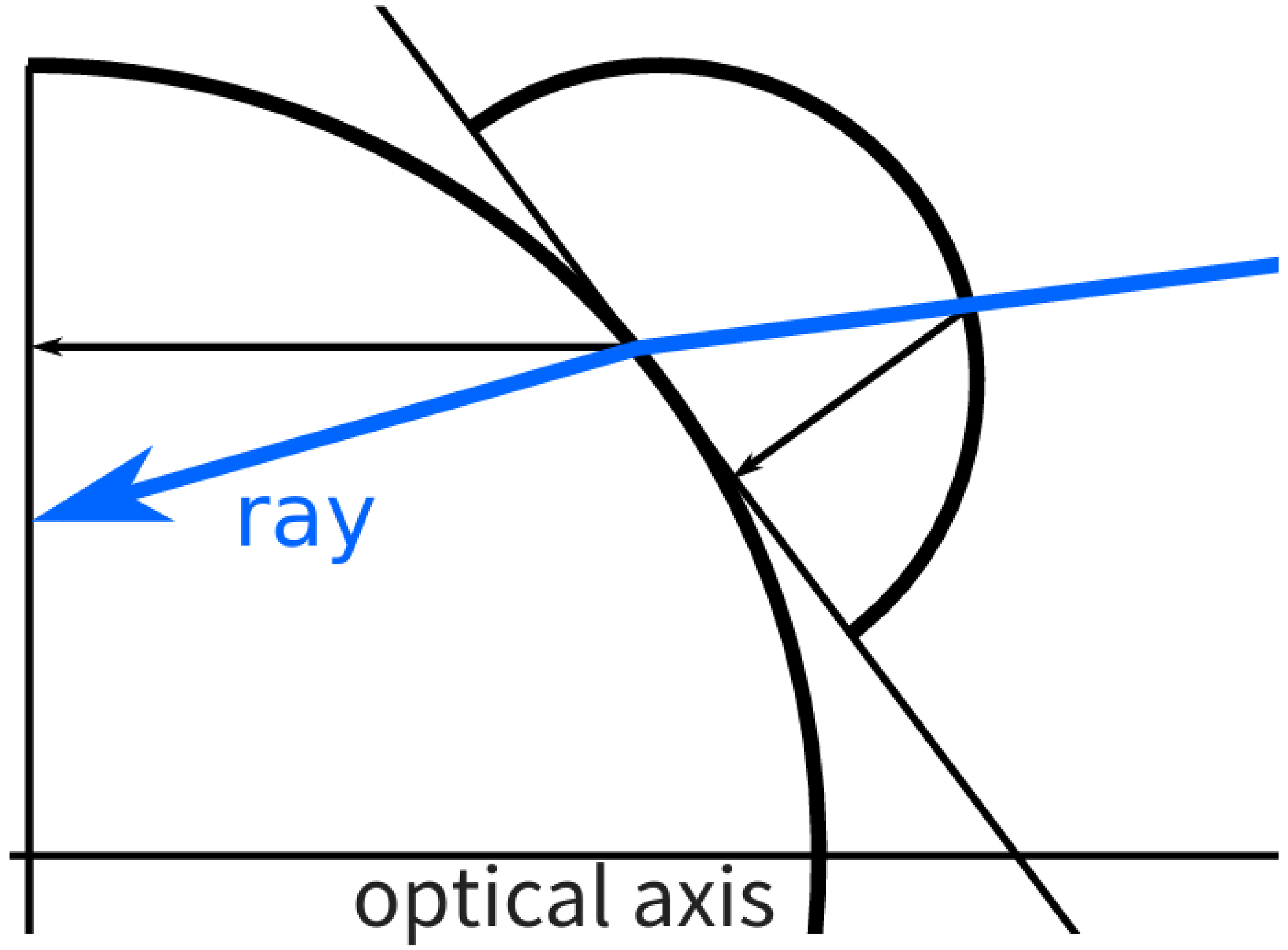

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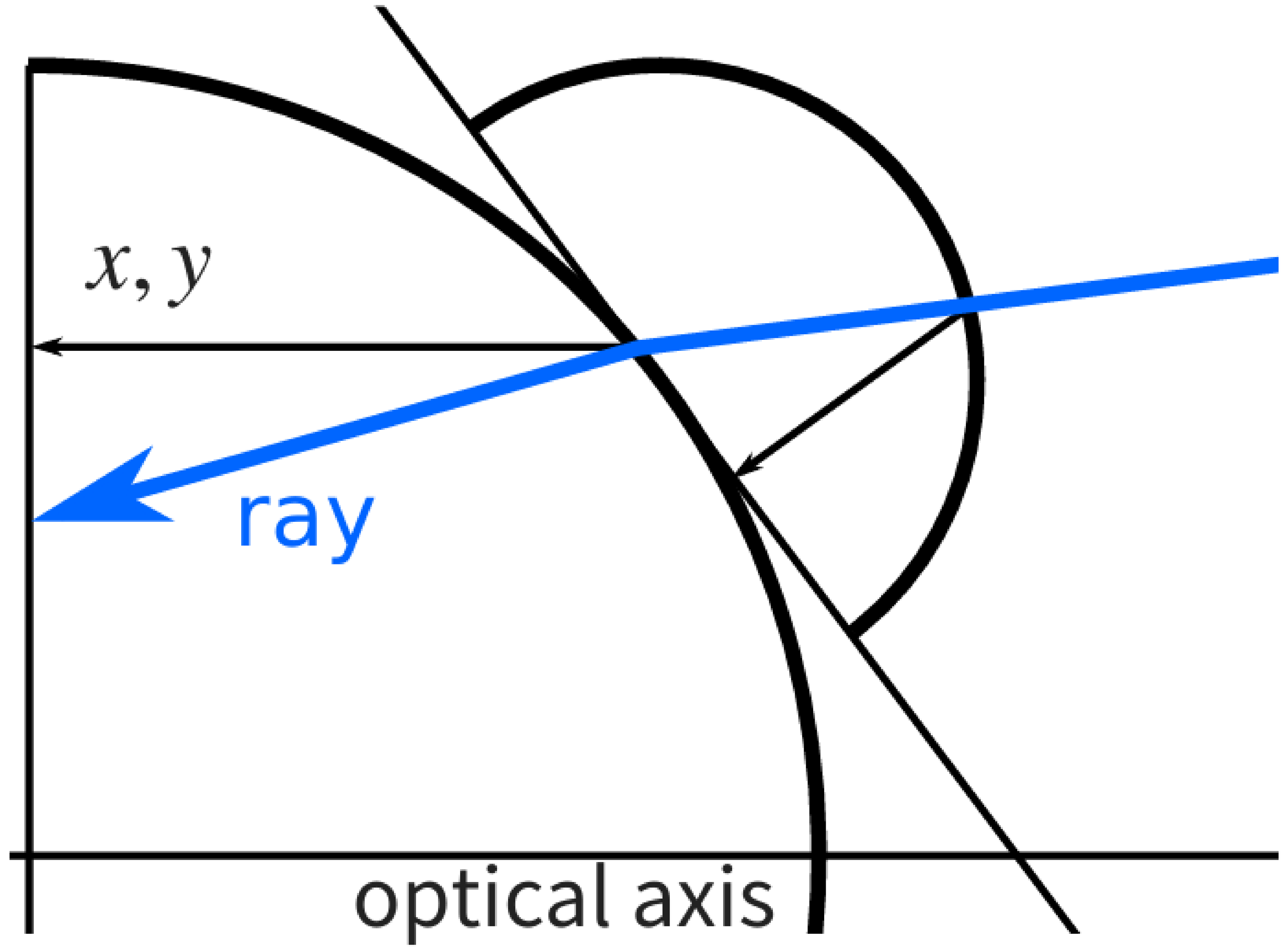

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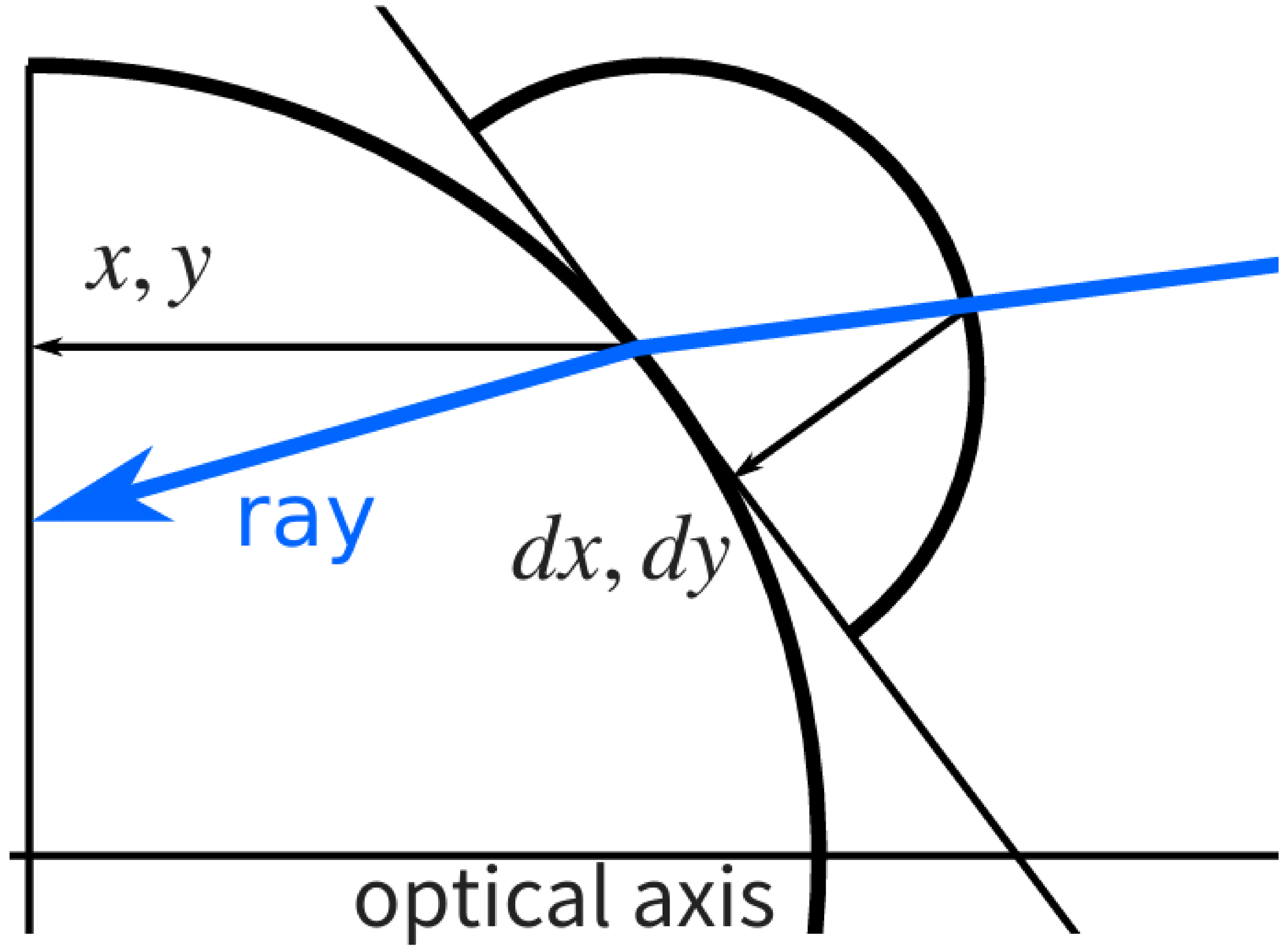

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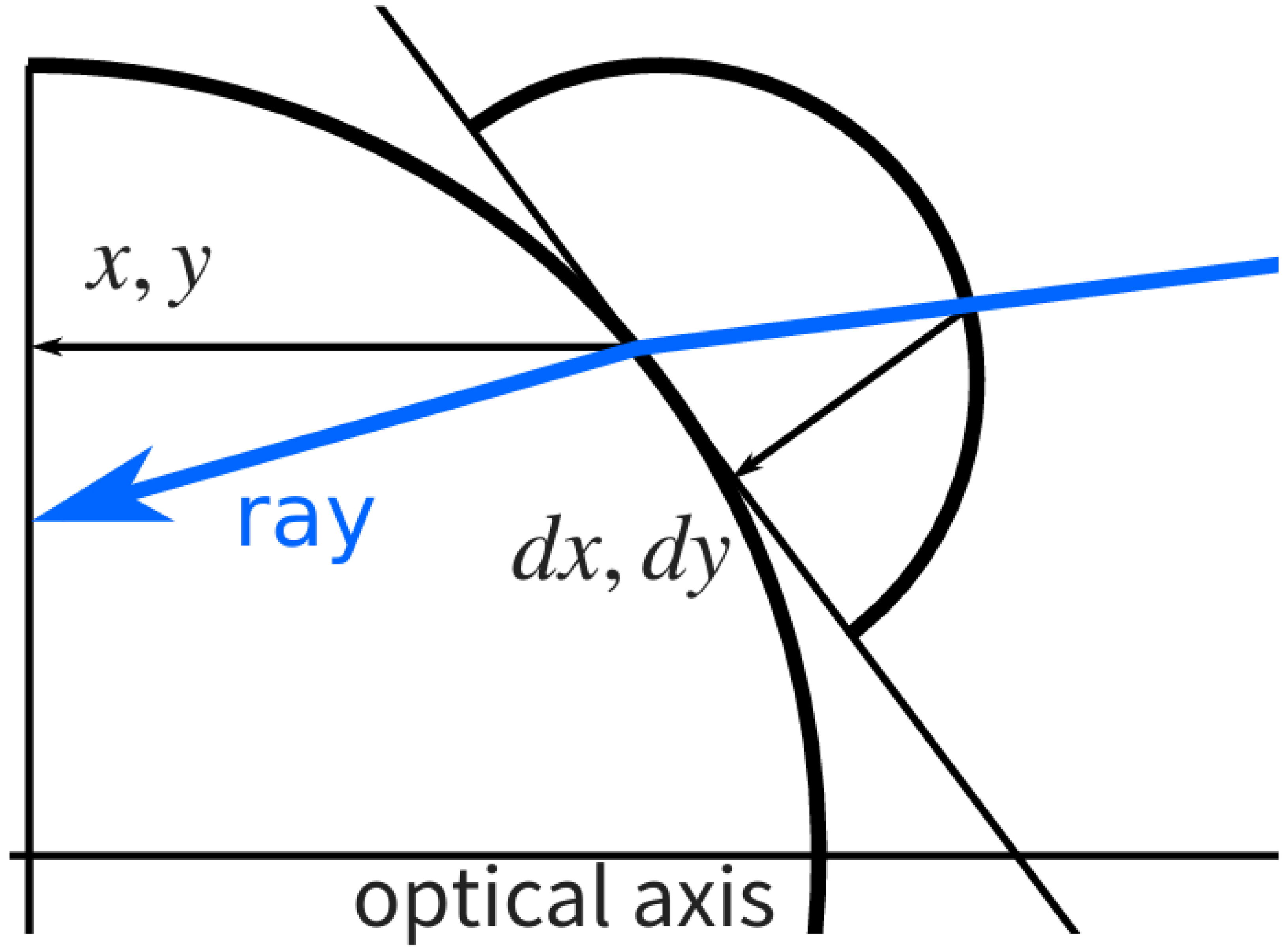

- need to specify tangent frame for dx, dy
- avoid the singularity in interesting regions on the outer pupil

parametrisation suitable for fisheye lenses

finding a polynomial

- polynomial consists of these terms:

$$c \cdot \underbrace{x_s^{d_0} y_s^{d_1} dx_s^{d_2} dy_s^{d_3} \lambda_s^{d_4}}_{=:T_t} \text{ with degree } \sum_{i=0}^{4} d_i \leq d$$

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- find most closely matching polynomial for given set of ray traced reference samples
  - linear problem, Galerkin projection of function $\mathbf{O} = P_o(\mathbf{S})$ to

$$\mathbf{O} \approx \hat{\Phi} \cdot \mathbf{c}$$

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- standard procedure (linear least squares), but the matrix is too large for our taste!

finding a *sparse* polynomial

- use *orthogonal matching pursuit* [TG07]
- iteratively select most important columns in $\hat{\Phi}$

$$\hat{\Phi} \cdot \mathbf{c} = \begin{pmatrix} T_1 & T_2 & T_3 & \cdots & T_{N-1} & T_N \\ T_1 & T_2 & T_3 & \cdots & T_{N-1} & T_N \\ \cdots & & & & & \\ T_1 & T_2 & T_3 & \cdots & T_{N-1} & T_N \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ . \\ . \\ c_{N-1} \\ c_N \end{pmatrix} \approx \mathbf{O}$$

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- original just looks for largest impact on residual (fast)
- we got better results by re-fitting all coefficents c of all previously selected columns in the inner loop (somewhat slower)
- details see the paper

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- we use up to 40 coefficients per equation (out of 4368 for degree 11)

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  - in particular no analytic Taylor expansion required!

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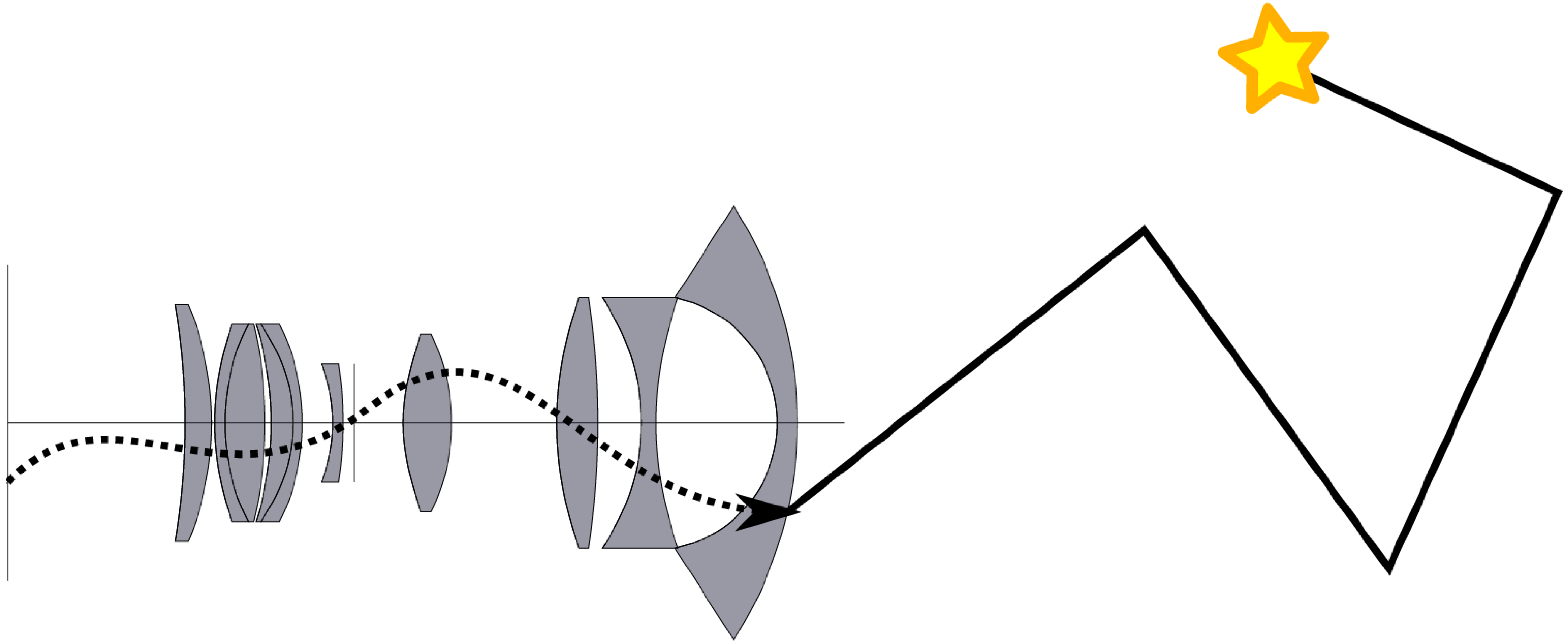
- we use up to 40 coefficients per equation (out of 4368 for degree 11)
- works transparently for aspheric and anamorphic lens elements
  - in particular no analytic Taylor expansion required!
- we also fit Fresnel transmittance τ to support coatings

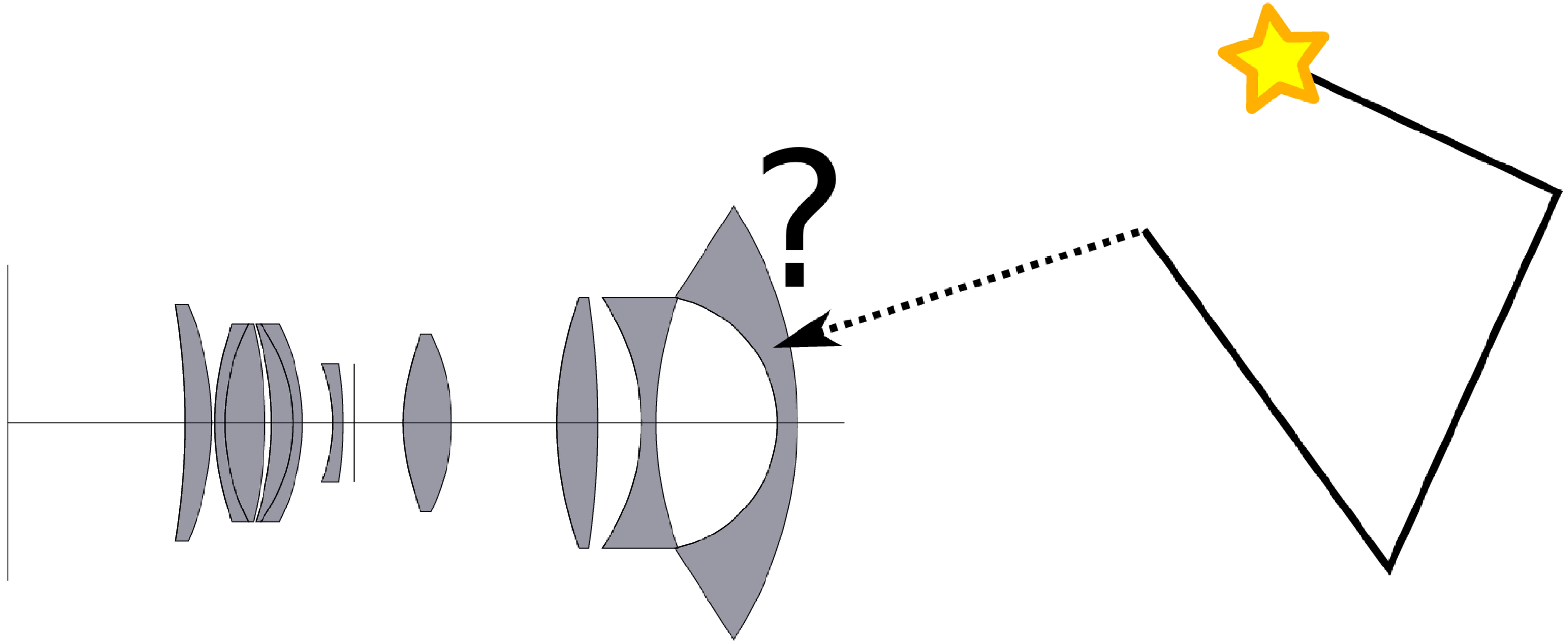
use polynomials for path tracing

- we know how do do this for path tracing from the camera
- efficiently by sampling the aperture [HD14]:
  - sample point on aperture
  - iteratively find position and direction on sensor
- require derivatives of polynomial
  - Newton's method



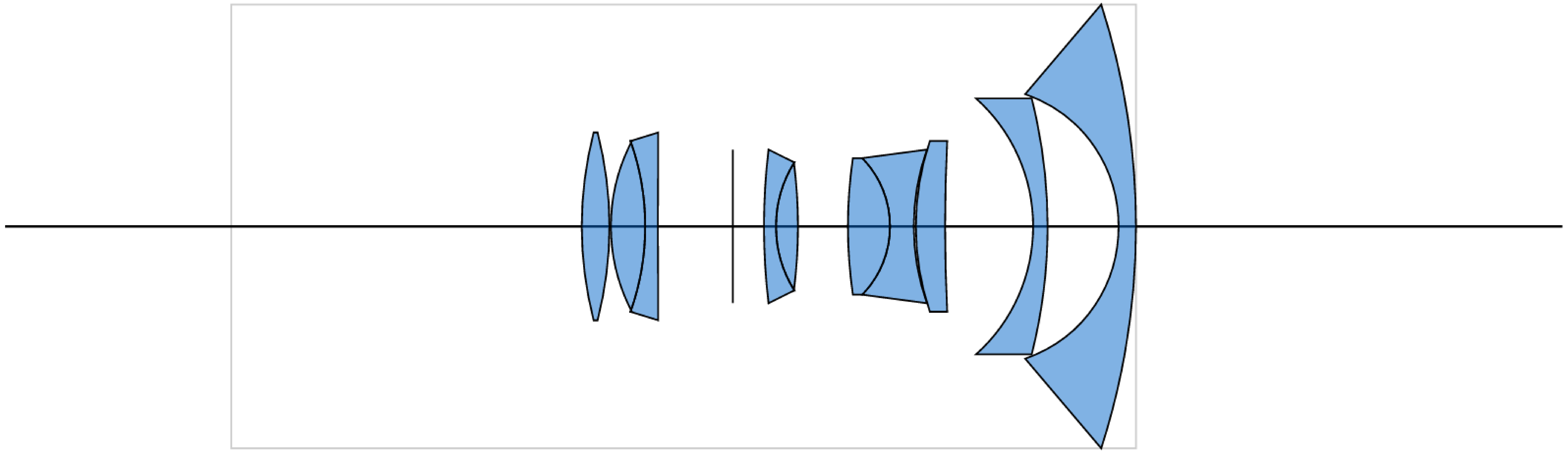
use polynomials for light tracing

- light tracing/deterministic camera connection?
  - sample point on aperture
  - keep point in scene fixed
  - iteratively find position and direction on sensor
- transform probability densities for multiple importance sampling
  - details see the paper



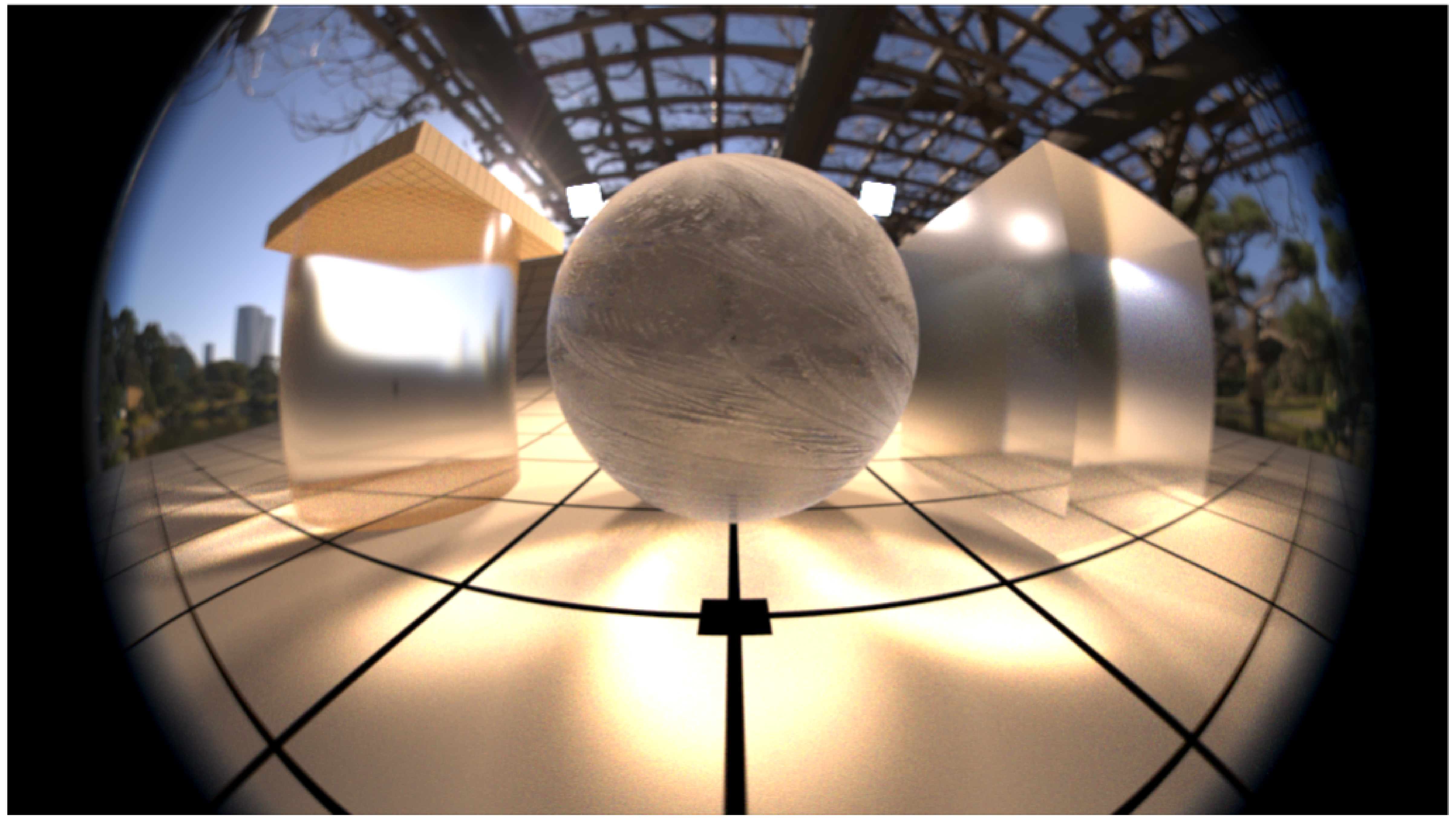
aperture sampling via 2-step Newton iteration

- initial guess: straight on optical axis
- aperture error ⇒ update sensor direction
- error in outgoing direction ⇒ update sensor position

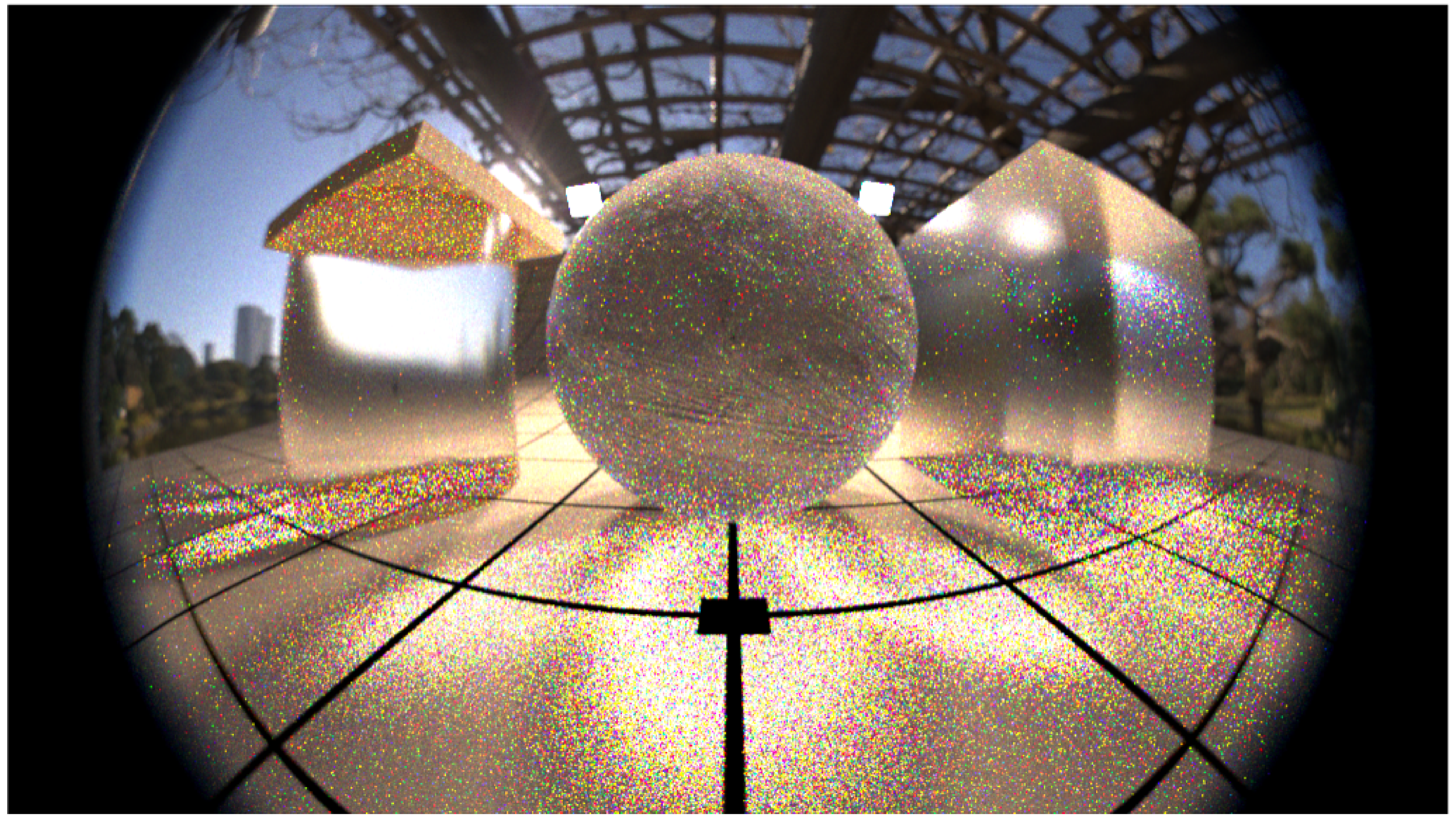


results: aperture sampling for light tracing (512spp)

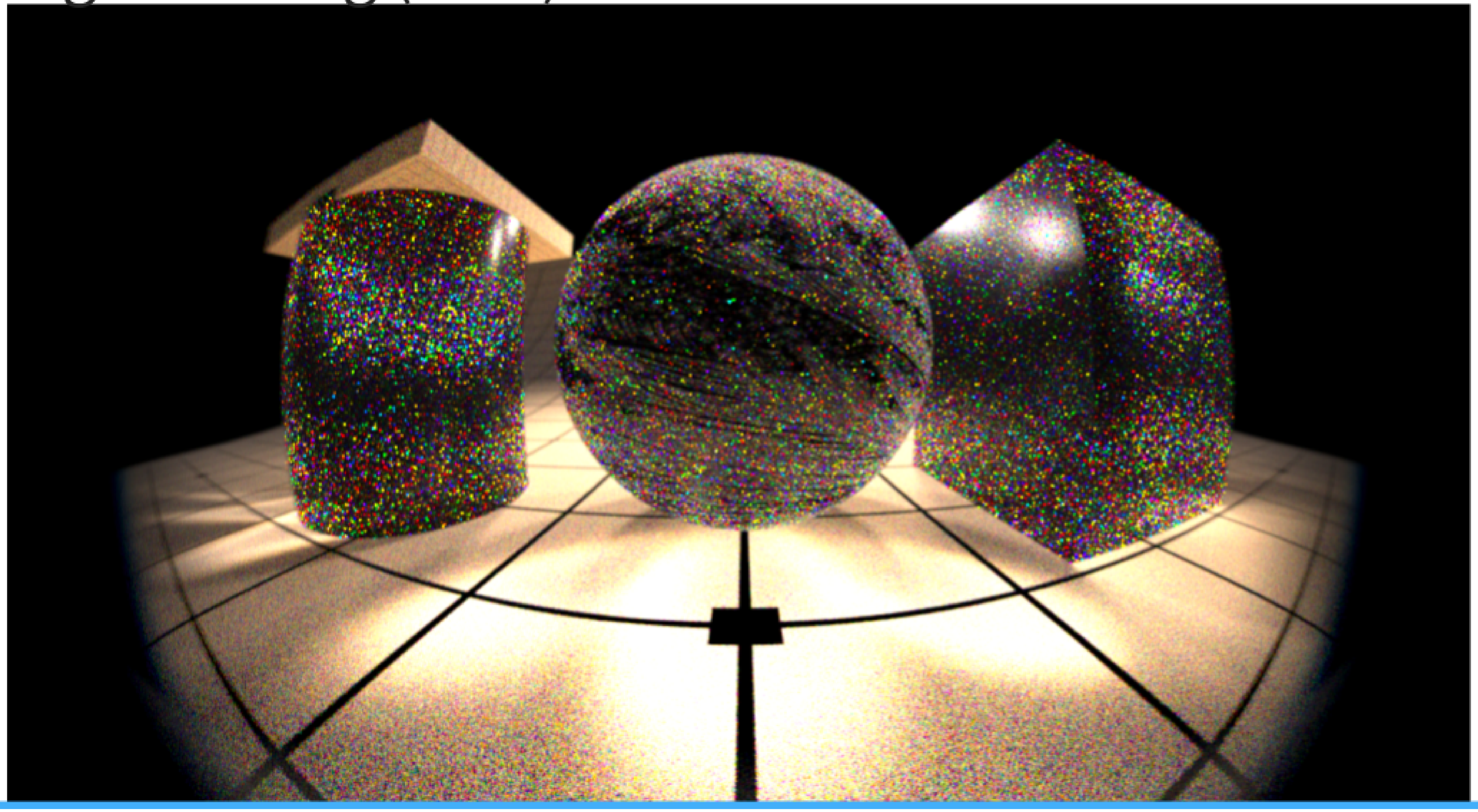
reference



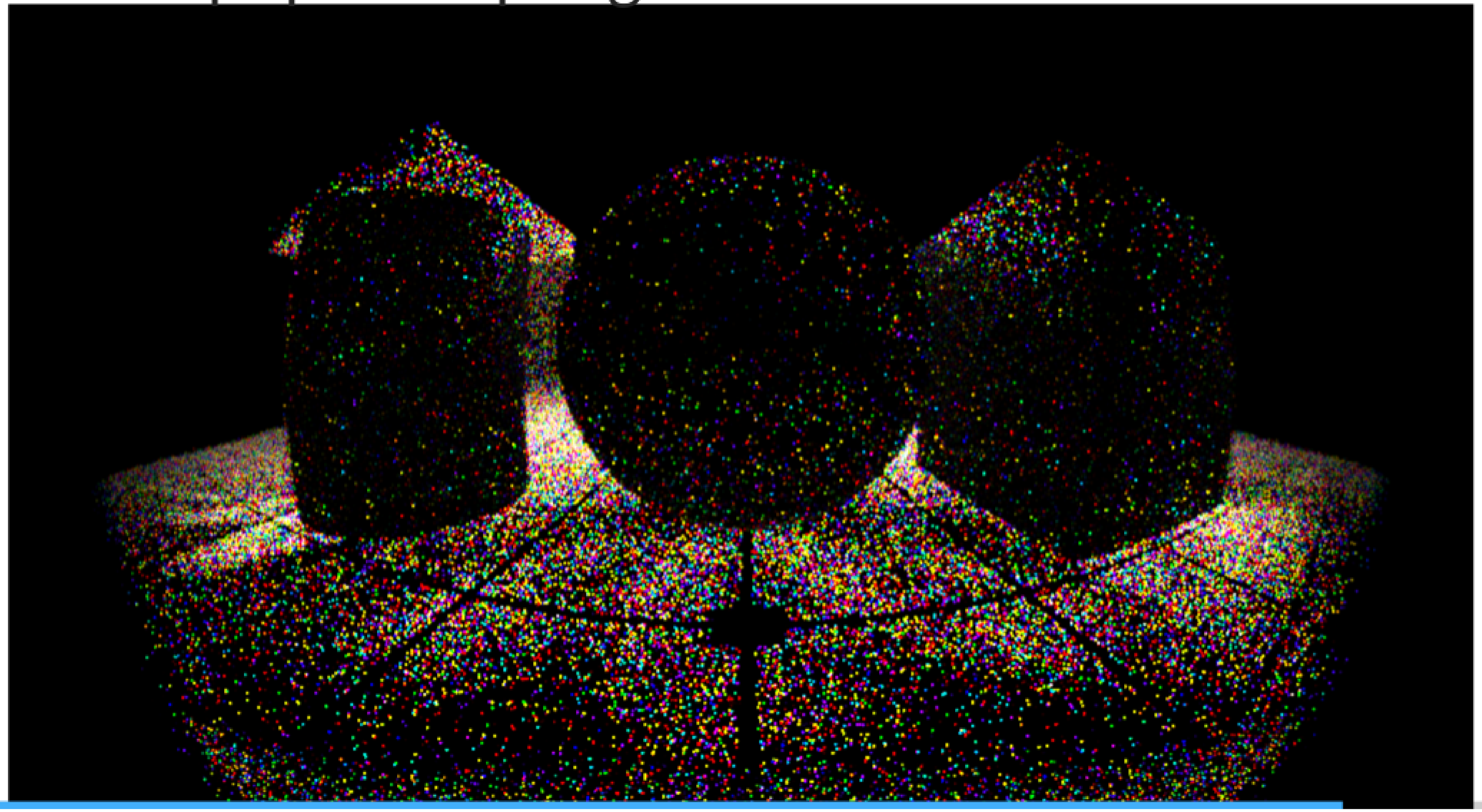
next event estimation



light tracing (ours)

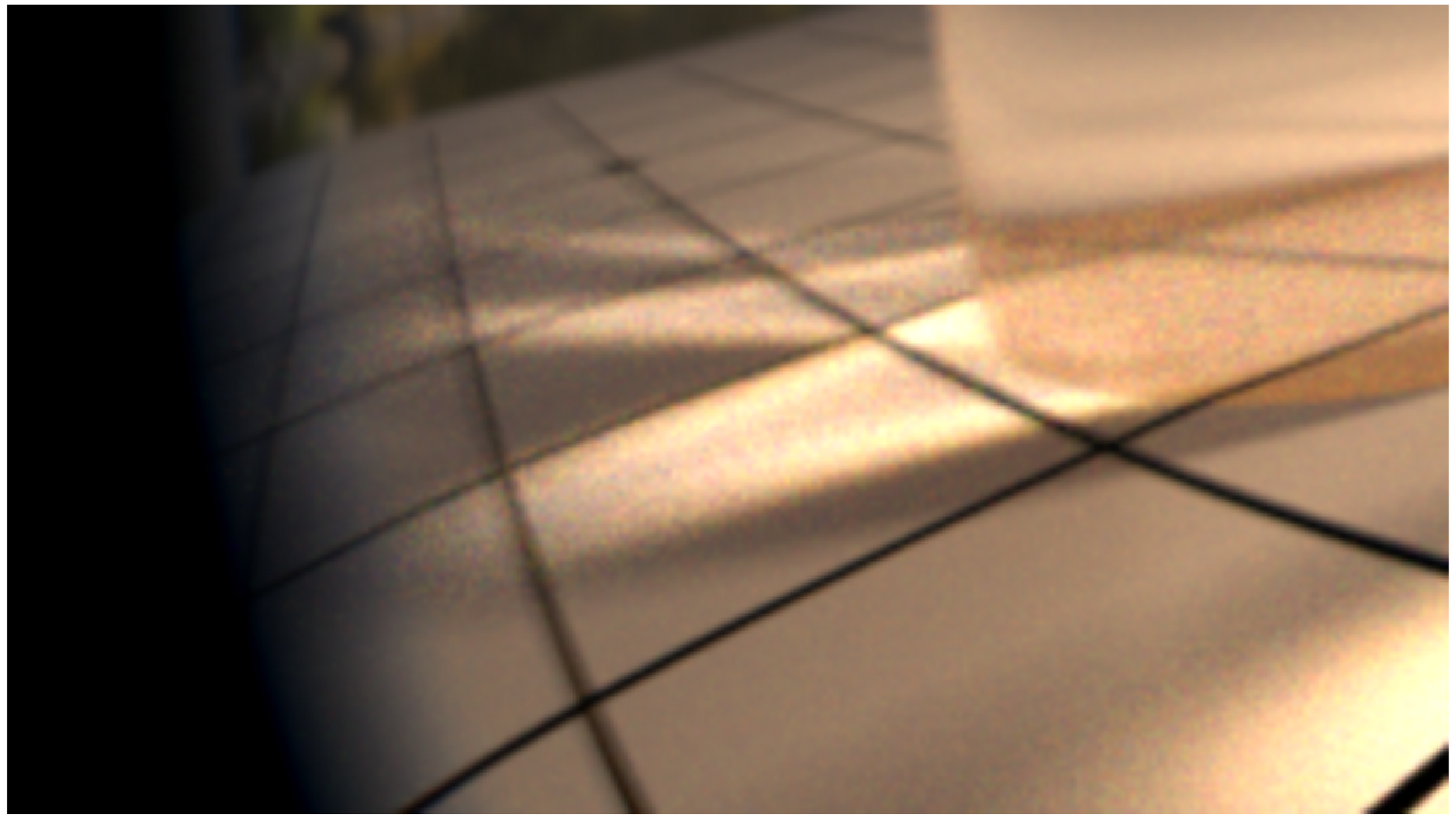


outer pupil sampling



results: aperture sampling for light tracing (closeup)

reference



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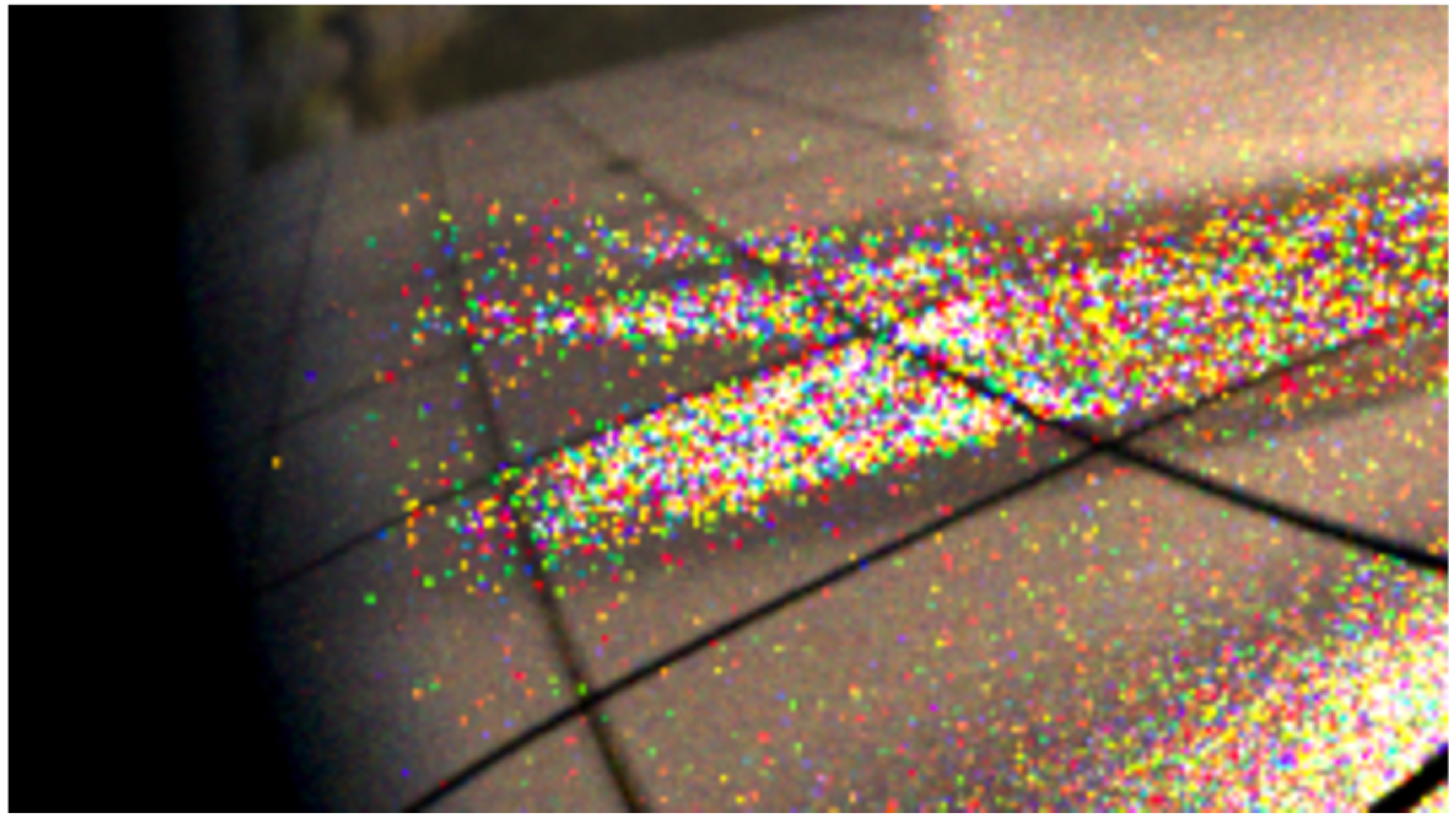

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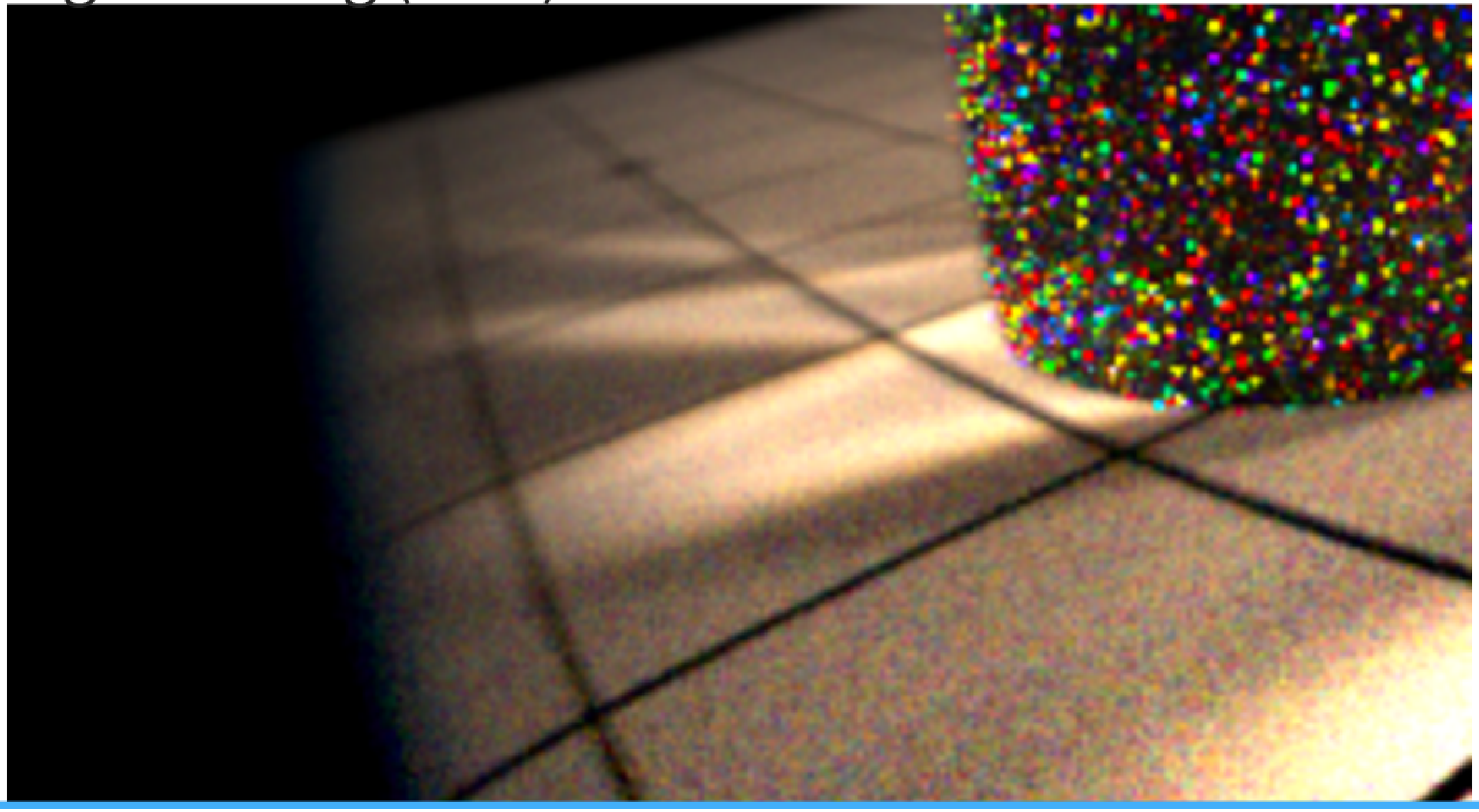

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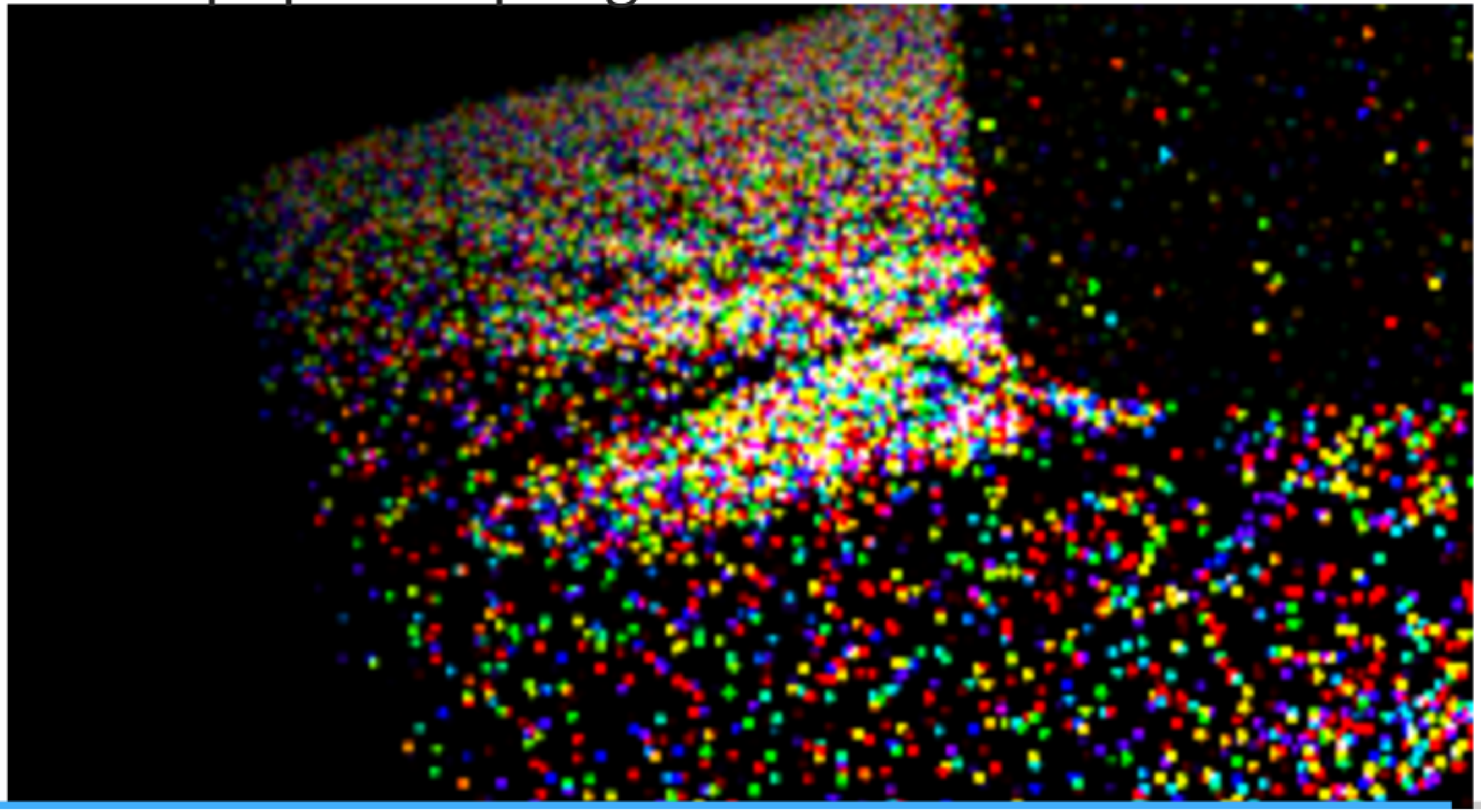
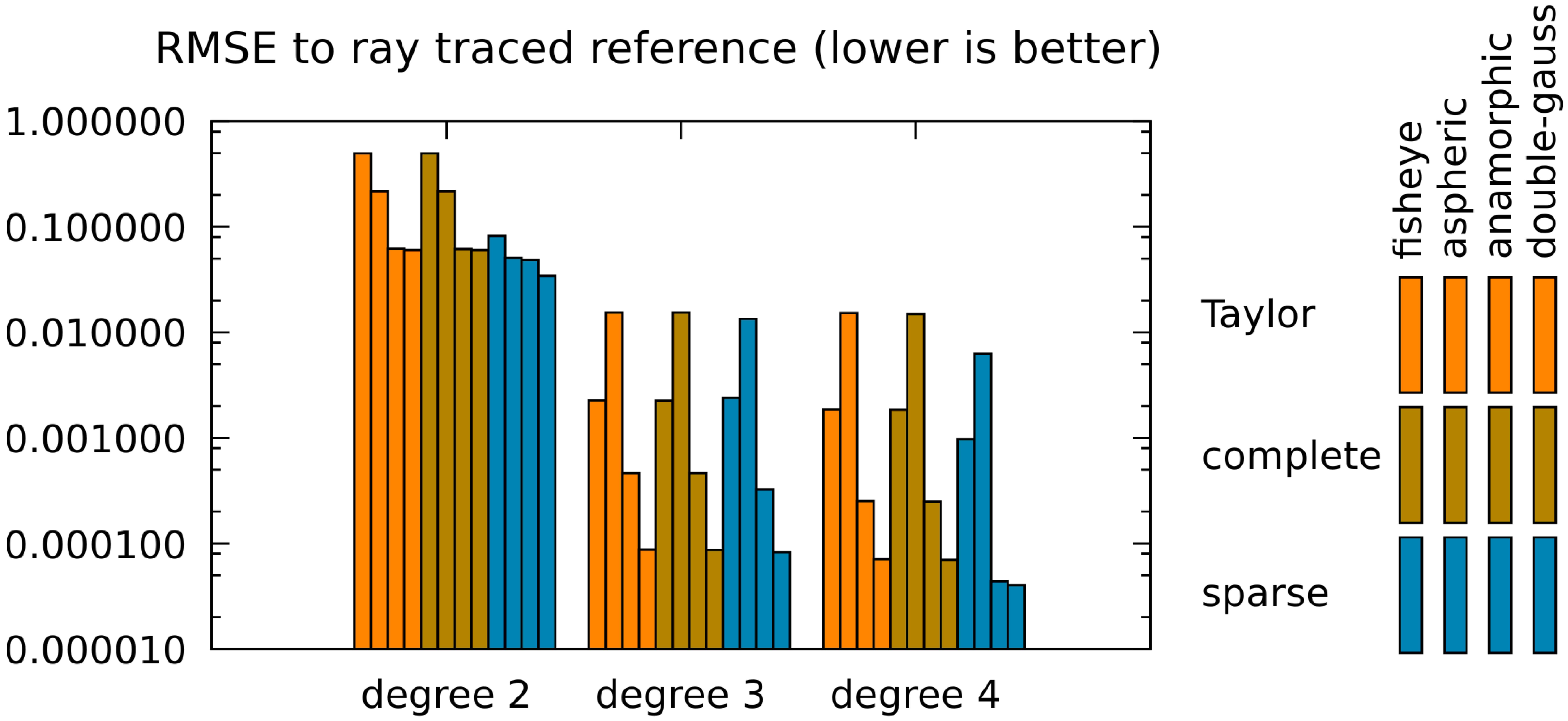


results: accuracy of sparse polynomials

- almost always better than Taylor or full polynomials (use higher degree terms!)
  - Taylor and complete: same degree (2, 3, 4)
  - Taylor and sparse: same number of coefficients
  - analytic Taylor expansion past degree 4 becomes very hard



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	coeffs	**fisheye**	**aspheric**	**anamorphic**	**double-gauss**
Taylor 2	4	4.97 · 10 −1	2.17 · 10 −1	6.18 · 10 −2	6.03 · 10 −2
Complete 2	21	4.97 · 10 −1	2.17 · 10 −1	6.17 · 10 −2	6.02 · 10 −2
Sparse	4	**8.21 · 10 −2**	**5.07 · 10 −2**	**4.85 · 10 −2**	**3.43 · 10 −2**
Taylor 3	16	2.26 · 10 −3	1.54 · 10 −2	4.63 · 10 −4	8.76 · 10 −5
Complete 3	56	**2.25 · 10 −3**	1.54 · 10 −2	4.62 · 10 −4	8.69 · 10 −5
Sparse	16	2.40 · 10 −3	**1.34 · 10 −2**	**3.25 · 10 −4**	**8.22 · 10 −5**
Taylor 4	28	1.86 · 10 −3	1.52 · 10 −2	2.52 · 10 −4	7.07 · 10 −5
Complete 4	126	1.85 · 10 −3	1.49 · 10 −2	2.50 · 10 −4	6.98 · 10 −5
Sparse	28	**9.72 · 10 −4**	**6.26 · 10 −3**	**4.40 · 10 −5**	**4.02 · 10 −5**

real time implementation

- works on deep image buffer data (here from [ZKP13])
- evaluate generated polynomial code in GLSL shader
- proof-of-concept implementation
  - 137 ms, 1080x720 px, 144 spp, AMD Radeon R9 390
  - limited by texture fetches more than by lens evaluation
- performance can probably be improved a lot by doing something smarter
  - e.g. Deferred Image-based Ray Tracing/HPG talk on Tuesday..
  - or with rasterisation (Comparison of Projection Methods for Rendering Virtual Reality)

conclusion

- more precise polynomials
  - higher degree terms, still sparse (fast)
- simpler construction
  - no Taylor expansion (which becomes untractable for higher degrees)
- now also practical for bidirectional/Metropolis
  - aperture sampling for light tracing
- proof of concept GPU implementation
- source code available

thank you for listening!

source code at https://jo.dreggn.org/home/2016_optics.tar.bz2